AN ESTIMATOR FOR AN ANTI-AIRCRAFT GUN FIRE CONTROL SYSTEM

John Michael Parr

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AN ESTIMATOR FOR AN ANTI-AIRCRAFT
GUN FIRE CONTROL SYSTEM

by

John Michael Parr

December 1974

Thesis Advisor: D. E. Kirk

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An Estimator for An Anti-Aircraft Gun Fire Control System

by

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I. INTRODUCTION

A. ALTERNATIVE ESTIMATION TECHNIQUES

Recently the Gun-Fire Control System MK 86 has been the subject of several studies done at the Naval Postgraduate School involving the use of Kalman filtering techniques in a gun-fire-control system [1],[2],[4]. This thesis examines some possible alternatives to the techniques used in GFCS MK 86 and makes comparisons to the GFCS MK 86 filter.

One proposed alternative to GFCS MK 86 procedure is the use of a rotated cartesian coordinate system. The coordinate system is rotated about the vertical (z) axis so that the x axis is aligned with the initial measured target bearing. Figure 1 illustrates the proposed coordinate system and its relationship to the fixed cartesian and spherical coordinate systems when noiseless measurements are assumed. In the general case, when the measurements are noisy, the x-axis will not be aligned exactly with the true target bearing as indicated in Fig. 1 but the error is expected to be small. The advantages gained by rotating the coordinate system are diminished correlation of the measurement noise and more accurate modeling of random forcing inputs. These attributes are discussed quantitatively in Chapter II.

In Chapter III estimators based on two models of target motion are developed: a constant-velocity model similar to one used in GFCS MK 86 [1] but utilizing the rotated coordinate system, and a constant-velocity model with a coloring

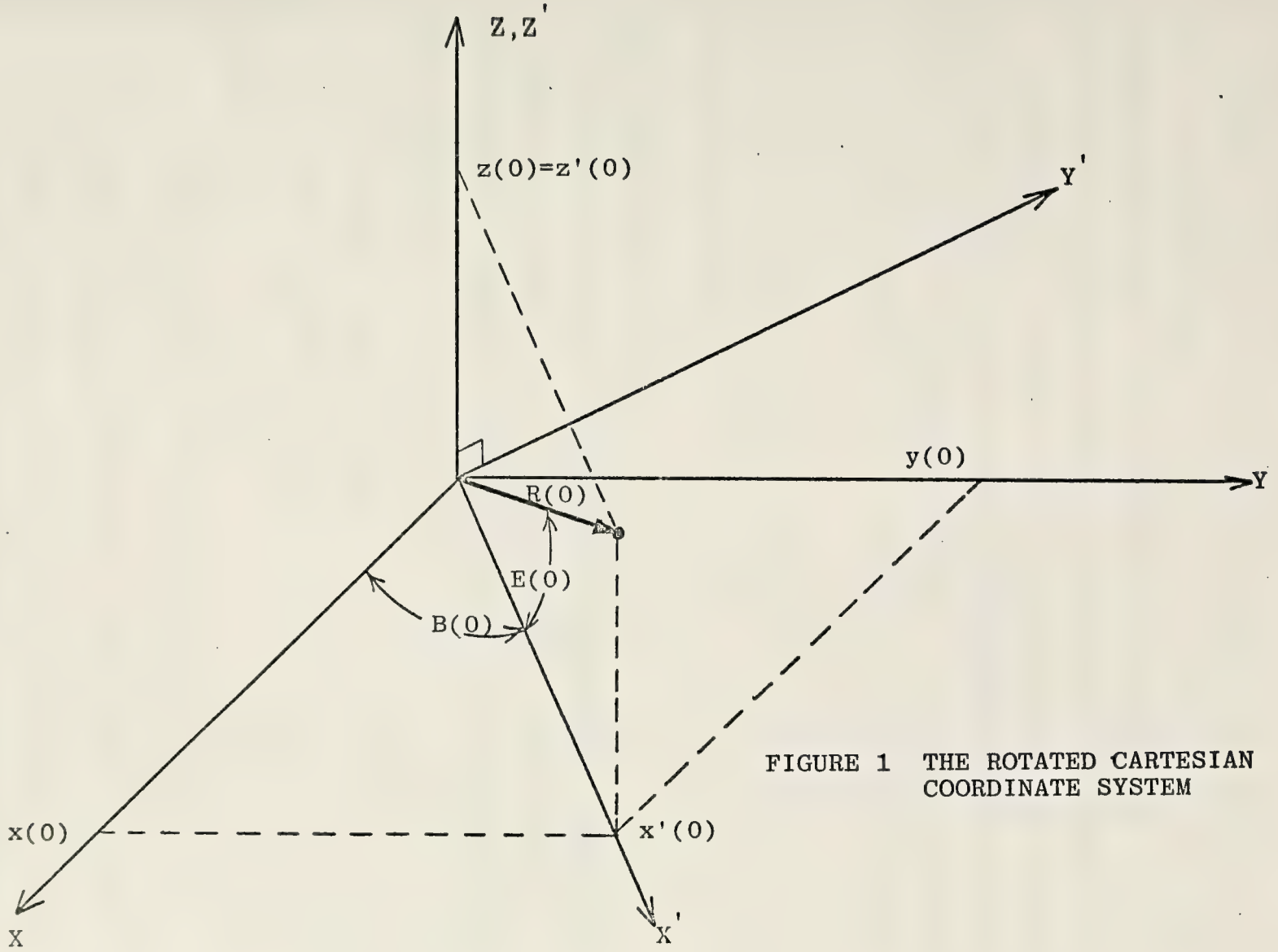

FIGURE 1 THE ROTATED CARTESIAN COORDINATE SYSTEM

pre-filter similar to that described by McKinley [4] also using the rotated coordinate system.

Monte-Carlo simulations were performed in order to evaluate the performance of several variations of the two basic filters developed in Chapter III. Descriptions and results of the Monte-Carlo simulations are presented in Chapter IV.

In Chapter V the results of the Monte-Carlo tests reported in Chapter IV are used to develop two methods of making the filter adaptive to changing situations. Monte-Carlo simulation results for the adaptive filters are reported in Chapter VI.

The final chapter summarizes the results of the various simulations and presents conclusions.

B. BACKGROUND MATERIAL

The model used for the system, composed of a target and a radar which makes noisy measurements of target range, bearing and elevation is approximated by the linear difference equations

$$\underset{\sim}{x}(k+1) = \underset{\sim\sim}{\Phi}\underset{\sim}{x}(k) = \underset{\sim\sim}{\Gamma}\underset{\sim}{w}(k) \tag{1.1}$$

$$\underset{\sim}{z}(k) = \underset{\sim\sim}{H}\underset{\sim}{x}(k) + \underset{\sim}{v}(k) \tag{1.2}$$

where

$\underset{\sim}{x}(k)$ is the n-dimensional state vector at time t=kT

$\underset{\sim}{w}(k)$ is the p-dimensional random forcing vector at time t=kT

$\underset{\sim}{\Phi}(k)$ is the n×n state transition matrix

$\underset{\sim}{\Gamma}(k)$ is the n×p matrix which related the random forcing terms to the state vector

$\underset{\sim}{z}(k)$ is the m-dimensional vector of measurements taken at time t=kT

$\underset{\sim}{H}$ is the m×n observation matrix

$\underset{\sim}{v}(k)$ is the m-dimensional vector of measurement noise

T is the sampling period

k is a non-negative integer which defines the current sample.

The assumed random process statistics are summarized by the following equations.

$$E\{\underset{\sim}{v}(k)\} = E\{\underset{\sim}{w}(k)\} = \underset{\sim}{0} \text{ for all } k \tag{1.3}$$

$$E\{\underset{\sim}{v}(k)\underset{\sim}{v}^T(j)\} = \underset{\sim}{R}(k)\delta_{jk} \tag{1.4}$$

$$\underset{\sim}{\Gamma}(k)E\{\underset{\sim}{w}(k)\underset{\sim}{w}^T(j)\}\Gamma^T(k) = \underset{\sim}{Q}(k)\delta_{jk} \tag{1.5}$$

$$E\{\underset{\sim}{v}(k)\underset{\sim}{w}(j)\} = \underset{\sim}{0} \text{ for all } j,k. \tag{1.6}$$

where

E is the expectation operator

δ_{jk} is the Kronecker delta function

$\underset{\sim}{R}(k)$ is the m×m covariance of measurement noise matrix

$\underset{\sim}{Q}(k)$ is the n×n state excitation covariance matrix.

The discrete Kalman filter equations are [5]

$$\underset{\sim}{G}(k) = \underset{\sim}{P}(k/k-1)\underset{\sim}{H}^T[\underset{\sim}{H}\underset{\sim}{P}(k/k-1)\underset{\sim}{H}^T + \underset{\sim}{R}(k)]^{-1} \tag{1.7}$$

$$\underset{\sim}{P}(k/k) = [\underset{\sim}{I} - \underset{\sim}{G}(k)\underset{\sim}{H}]\underset{\sim}{P}(k/k-1) \tag{1.8}$$

$$\underset{\sim}{P}(k/k-1) = \underset{\sim}{\Phi}\underset{\sim}{P}(k-1/k-1)\underset{\sim}{\Phi}^T + \underset{\sim}{Q}(k-1) \tag{1.9}$$

$$\hat{\underset{\sim}{X}}(k/k-1) = \underset{\sim}{\Phi}\hat{\underset{\sim}{X}}(k-1/k-1) \tag{1.10}$$

$$\hat{\underset{\sim}{X}}(k/k) = \hat{\underset{\sim}{X}}(k/k-1) + \underset{\sim}{G}(k)[\underset{\sim}{Z}(k) - \underset{\sim}{H}\hat{\underset{\sim}{X}}(k/k-1)] \tag{1.11}$$

where the notation (k/k-1) denotes the calculation of a quantity at time t=kT based on information up to and

including time t=(k-1)T. The previously undefined matrices which appear in the Kalman filter equations are defined as follows:

$\underset{\sim}{G}(k)$ - the n×m Kalman filter gain matrix,

$\underset{\sim}{I}$ - the n×n identity matrix,

$\underset{\sim}{P}(k/k)$ - the n×n covariance of estimation error matrix,

$\underset{\sim}{P}(k/k-1)$ - the one-sample prediction error covariance matrix,

$\hat{\underset{\sim}{X}}(k/k)$ - the n×1 state estimate vector,

$\hat{\underset{\sim}{X}}(k/k-1)$ - the n×1 state prediction vector.

II. THE ROTATED CARTESIAN COORDINATE SYSTEM

A. MOTIVATION FOR ROTATING THE COORDINATE SYSTEM

1. Correlation of Measurement Noise

When radar measurements taken in spherical coordinates are converted to cartesian coordinates for filter processing the measurement noise becomes correlated in the process.

The measurements in spherical coordinates are given by

$$Z_1(k) = r(k) + n_1(k) \tag{2.1}$$

$$Z_2(k) = B(k) + n_2(k) \tag{2.2}$$

$$Z_3(k) = E(k) + n_3(k) \tag{2.3}$$

where $r(k)$, $B(k)$ and $E(k)$ are true target range, bearing and elevation respectively and $n_1(k)$, $n_2(k)$ and $n_3(k)$ are the corresponding measurement noise components. It is assumed that $n_1(k)$, $n_2(k)$ and $n_3(k)$ are mutually uncorrelated and that

$$E\{n_1(k)\} = E\{n_2(k)\} = E\{n_3(k)\} = 0$$

$$E\{n_1^2(k)\} = \sigma_r^2,\ E\{n_2^2(k)\} = \sigma_B^2,\ E\{n_3^2(k)\} = \sigma_E^2\ ,$$

$$k = 0,1,2,3,.....$$

If the coordinate converted measurements are written as

$$Z_1'(k) = x(k) + v_1(k)$$

$$Z_2'(k) = y(k) + v_2(k)$$

$$Z_3'(k) = z(k) + v_3(k)$$

it can be shown that [2]

$$E\{\underset{\sim}{v}(k)\} = \underset{\sim}{0}$$

$$E\{\underset{\sim}{v}(k)\underset{\sim}{v}^T(k)\} = \underset{\sim}{R}(k) \tag{2.4}$$

where the entries of the covariance matrix of measurement error $\underset{\sim}{R}(k)$ are

$$\begin{aligned} R_{11}(k) = {} & r^2(k)\sigma_E^2\cos^2B(k)\sin^2E(k) \\ & + r^2(k)\sigma_B^2\sin^2B(k)\cos^2E(k) \\ & + r^2(k)\sigma_B^2\sigma_E^2\sin^2B(k)\sin^2E(k) \\ & + \sigma_r^2\cos^2B(k) \end{aligned} \tag{2.5}$$

$$\begin{aligned} R_{12}(k) = {} & r^2(k)\cos B(k)\sin B(k)\sin^2E(k)[\sigma_E^2-\sigma_B^2\sigma_E^2] \\ & + \sin B(k)\cos B(k)\cos^2E(k)[\sigma_r^2-r^2(k)\sigma_B^2] \end{aligned} \tag{2.6}$$

$$R_{13}(k) = \cos B(k)\sin E(k)\cos E(k)[\sigma_r^2-r^2(k)\sigma_E^2] \tag{2.7}$$

$$R_{21}(k) = R_{12}(k) \tag{2.8}$$

$$\begin{aligned} R_{22}(k) = {} & r^2(k)\sigma_E^2\sin^2B(k)\sin^2E(k) \\ & + r^2(k)\sigma_B^2\cos^2B(k)\cos^2E(k) \\ & + r^2(k)\sigma_B^2\sigma_E^2\cos^2B(k)\sin^2E(k) \\ & + \sigma_r^2\sin^2B(k)\cos^2E(k) \end{aligned} \tag{2.9}$$

$$R_{23}(k) = \sin B(k)\sin E(k)\cos E(k)[\sigma_r^2-r^2(k)\sigma_E^2] \tag{2.10}$$

$$R_{31}(k) = R_{13}(k) \tag{2.11}$$

$$R_{32}(k) = R_{23}(k) \tag{2.12}$$

$$R_{33}(k) = r^2(k)\sigma_E^2\cos^2E(k) + \sigma_r^2\sin^2E(k) \tag{2.13}$$

The GFCS MK 86 filter assumes zero mean uncorrelated noise [1], however, equations (2.5) through (2.13) indicate that in general the measurement noise is correlated.

The effect of the correlation is diminished in the rotated cartesian coordinate system, since for an inbound target the bearing will be zero or near zero. If B(k) = 0 is assumed,

$$\underset{\sim}{R}(k) = \begin{bmatrix} r^2(k)\sigma_E^2 \sin^2 E(k) + \sigma_r^2 & 0 & \sin E(k)\cos E(k)[\sigma_r^2 - r^2(k)\sigma_E^2] \\ 0 & r^2(k)\sigma_B^2[\cos^2 E(k) + \sigma_E^2 \sin^2 E(k)] & 0 \\ \sin E(k)\cos E(k)[\sigma_r^2 - r^2(k)\sigma_E^2] & 0 & r^2(k)\sigma_E^2 \cos^2 E(k) + \sigma_r^2 \sin^2 E(k) \end{bmatrix} \tag{2.14}$$

Although the measurement noise is still correlated, the dependence on target bearing has been eliminated. For the case of the low elevation target, where the measurement noise is expected to be relatively high due to poor tracking conditions in the radar, the approximations $\sin E(k) \simeq 0$ and $\cos E(k) \simeq 1$ are valid and

$$\underset{\sim}{R}(k) \simeq \begin{bmatrix} \sigma_r^2 & 0 & 0 \\ 0 & r^2(k)\sigma_B^2 & 0 \\ 0 & 0 & r^2(k)\sigma_E^2 \end{bmatrix} \tag{2.15}$$

which agrees in form with the uncorrelated noise model.

For the fixed coordinate system the low elevation assumption yields

$$\underset{\sim}{R}(k) = \begin{bmatrix} r^2(k)\sigma_B^2\sin^2B(k) & \sin B(k)\cos B(k)[\sigma_r^2 & 0 \\ +\sigma_r^2\cos^2B(k) & -r(k)\sigma_B^2] & \\ & r^2(k)\sigma_B^2\cos^2B(k) & \\ R_{12}=R_{21} & +\sigma_r^2\sin^2B(k) & 0 \\ 0 & 0 & r^2(k)\sigma_E^2 \end{bmatrix} \quad (2.16)$$

which agrees with the uncorrelated noise model only when B(k) = 0, 90, 180, or 270 degrees. Equations (2.15) and (2.16) indicate that the uncorrelated measurement noise assumption is likely to be considerably more accurate in the rotated coordinate system than it is in the fixed cartesian coordinate system.

2. Modeling of Random Forcing Inputs

The relative magnitudes of the vector components of forcing inputs in fixed cartesian coordinates depend on the target's orientation within the coordinate system.

The track generating program described in Appendix C has been used to demonstrate the change in the relative magnitude of the x and y acceleration components of a target maneuvering in the xy plane as the bearing of the track's axis of symmetry is changed as illustrated in Fig. 2 for two bearings. Inbound trajectories with seven-g maneuvers in the xy plane were generated using the track generating program in ten separate runs with each run differing only in the bearing angle of the axis of symmetry of the trajectory.

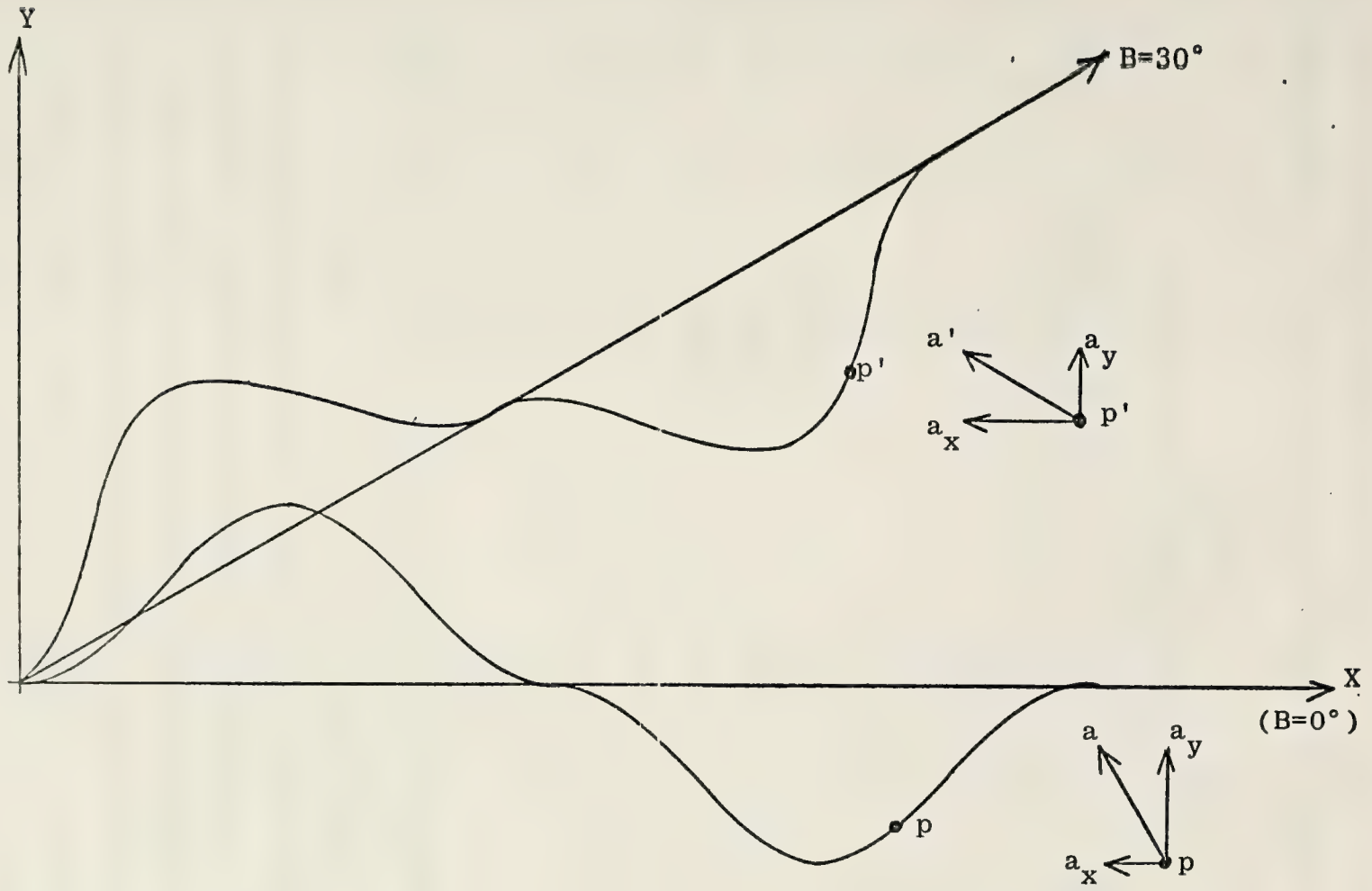

FIGURE 2 THE ROTATED XY PLANE MANEUVER SHOWING ACCELERATION COMPONENTS AT POINT P (BEFORE ROTATION) AND POINT P' (AFTER ROTATION)

Table I shows the relationship between the maximum acceleration in the x-direction, $|\ddot{x}|_{max}$, and the maximum acceleration in the y-direction, $|\ddot{y}|_{max}$, for each run.

TABLE I

COMPARISON OF MAXIMUM ACCELERATIONS

RUN	BEARING OF AXIS OF SYMMETRY (deg)	$\lvert\ddot{x}\rvert_{max}$(g)	$\lvert\ddot{y}\rvert_{max}$(g)
1	0	0.6	7.0
2	10	1.4	6.9
3	20	2.6	6.7
4	30	3.6	6.1
5	40	4.7	5.4
6	50	5.5	4.6
7	60	6.2	3.5
8	70	6.8	2.5
9	80	6.9	1.4
10	90	7.0	0.7

The large variation in the relative magnitudes of the acceleration components indicates that much more accurate apriori estimates of random forcing inputs are possible if all targets approach the origin from the same bearing. Therefore a considerable advantage in filter design might be gained by rotating the coordinate system so that the x-axis is aligned with the initial detection bearing of each new target.

B. COORDINATE SYSTEM ROTATION

The radar measurements of target position are range, r(k), bearing, B(k), and elevation, E(k) (to simplify the notation in the subsequent discussion r(k), B(k), and E(k) are assumed to include the measurement noise). To convert the measurements to cartesian coordinates the equations are

$$x(k) = r(k)\cos B(k)\cos E(k) \tag{2.17}$$

$$y(k) = r(k)\sin B(k)\cos E(k) \tag{2.18}$$

$$z(k) = r(k)\sin E(k) \tag{2.19}$$

where once again for simplicity of notation x(k), y(k) and z(k) are assumed to include measurement noise.

If the x-axis is aligned with the initial bearing of the target, equations (2.17) through (2.19) become

$$x'(k) = r(k)\cos\Delta B(k)\cos E(k) \tag{2.20}$$

$$y'(k) = r(k)\sin\Delta B(k)\cos E(k) \tag{2.21}$$

$$z'(k) = r(k)\sin E(k) \tag{2.22}$$

where

$$\Delta B(k) \equiv B(k) - B(0) \tag{2.23}$$

as depicted in Fig. 3.

In order to maintain the orientation of the rotated coordinate system it is necessary to store the initial bearing, B(0), for use in the coordinate conversion process as indicated in equation (2.23).

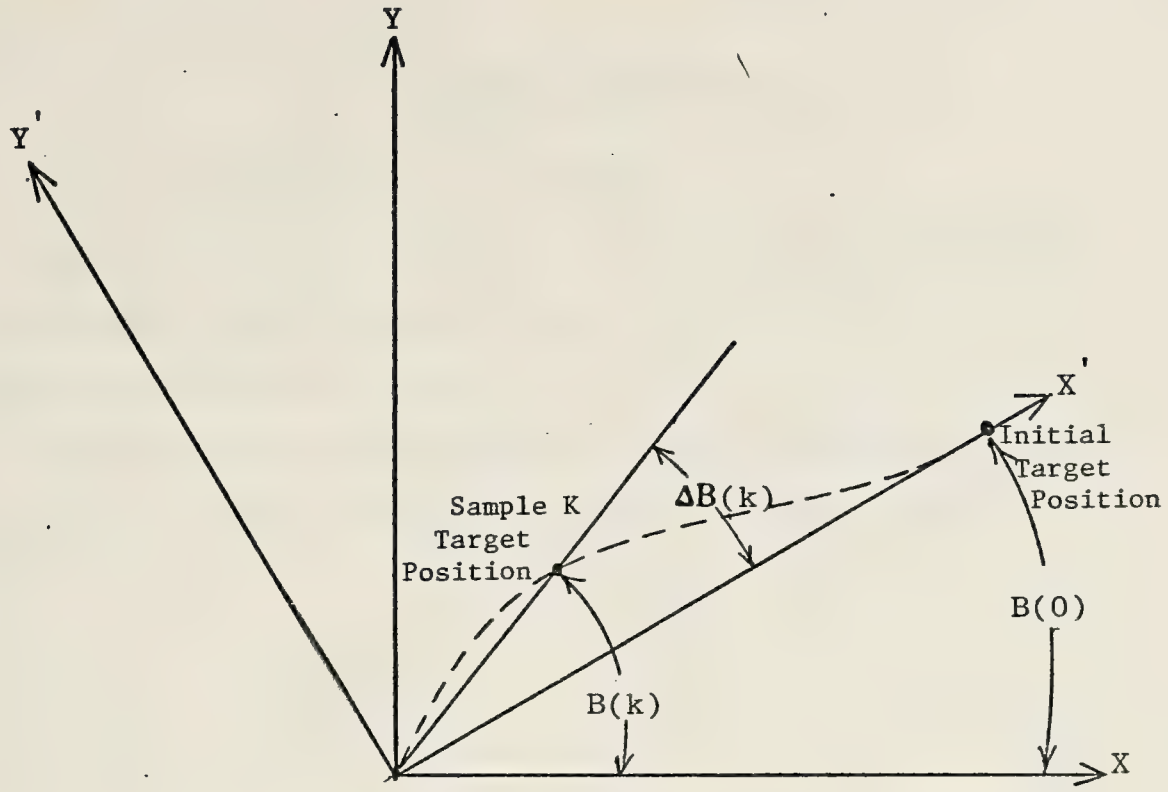

FIGURE 3 ROTATED COORDINATE SYSTEM GEOMETRY IN RELATION TO FIXED CARTESIAN COORDINATES

(Horizontal Plane Shown-Z Axis is Perpendicular to the Page)

III. FILTER DEVELOPMENT

A. THE CONSTANT-VELOCITY ($1/S^2$) MODEL

The first filter considered is based on a model which assumes that the target maintains constant velocity in all three coordinate directions.

The state transition matrix, $\underset{\sim}{\Phi}$, and $\underset{\sim}{\Gamma}$, the matrix which relates the random forcing inputs to the state vector are [1]

$$\underset{\sim}{\Phi}(T) = \left[\begin{array}{cc|cc|cc} 1 & T & & & & \\ 0 & 1 & & \underset{\sim}{0} & & \underset{\sim}{0} \\ \hline & & 1 & T & & \\ & \underset{\sim}{0} & 0 & 1 & & \underset{\sim}{0} \\ \hline & & & & 1 & T \\ & \underset{\sim}{0} & & \underset{\sim}{0} & 0 & 1 \end{array}\right] \tag{3.1}$$

and

$$\underset{\sim}{\Gamma}(T) = \left[\begin{array}{cc|cc|cc} T^2/2 & & & & & \\ T & & & \underset{\sim}{0} & & \underset{\sim}{0} \\ \hline & & T^2/2 & & & \\ & \underset{\sim}{0} & T & & & \underset{\sim}{0} \\ \hline & & & & T^2/2 & \\ & \underset{\sim}{0} & & \underset{\sim}{0} & T & \end{array}\right] \tag{3.2}$$

for the case of the constant-velocity system model with sampling time T.

The covariance of measurement noise matrix $\underset{\sim}{R}(k)$, for the system is given in Chapter II, equation (2.14). If the filter gains are to be calculated off-line, a model must be chosen for $\underset{\sim}{R}(k)$. This necessitates that values be chosen

for target range, target elevation, and the standard deviations of measurement noise. Range and elevation values were arbitrarily chosen as

$$r(k) = 10 \text{ kyd} \tag{3.3}$$

$$E(k) = 0 \text{ deg} \tag{3.4}$$

the measurement noise standard deviations used were

$$\sigma_r = 3 \text{ yd} \tag{3.5}$$

$$\sigma_B = \sigma_E = 0.3 \text{ mrad} \tag{3.6}$$

These values of σ_r, σ_B, and σ_E are considered to be typical of good tracking radar systems. Substituting the values given in equations (3.3) through (3.6) into equation (2.14) yields

$$\underset{\sim}{R}(k) = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \tag{3.7}$$

The system model described above is highly simplified. The model allows only for targets which fly at constant velocity. It is not expected that any target which the system might be used against would behave so well. Almost certainly there will be pilot initiated accelerations, wind buffeting, etc. which are not accounted for in the model. It is the function of the state excitation covariance matrix $\underset{\sim}{Q}$ to compensate for the inadequacies of the simplified model.

The starting point for developing the state excitation covariance matrix is the random forcing input vector, $\underset{\sim}{w}$, represented by

$$\underset{\sim}{w} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \tag{3.8}$$

where a_x, a_y, and a_z are accelerations in the x, y, and z-coordinate directions respectively. Since any coupling between the accelerations depends on the type of random forcing experienced, it is assumed for this model that the random forcing is decoupled, i.e.

$$E\{a_x a_y\} = E\{a_x a_z\} = E\{a_y a_z\} = 0$$

The state excitation matrix $\underset{\sim}{Q}$ is defined in Chapter I as

$$\underset{\sim}{Q} = E\{\underset{\sim}{\Gamma}\underset{\sim}{w}\underset{\sim}{w}^T\underset{\sim}{\Gamma}^T\} \tag{3.9}$$

If the matrix multiplication is performed with $\underset{\sim}{\Gamma}$ and $\underset{\sim}{w}$ as previously defined, and the expectation operation performed on the product, the result is

$$\underset{\sim}{Q} = \left[\begin{array}{cc|cc|cc} \frac{a_x^2T^4}{4} & \frac{a_x^2T^3}{2} & & & & \\ \frac{a_x^2T^3}{2} & a_x^2T^2 & \multicolumn{2}{c|}{\underset{\sim}{0}} & \multicolumn{2}{c}{\underset{\sim}{0}} \\ \hline & & \frac{a_y^2T^4}{4} & \frac{a_y^2T^3}{2} & & \\ \multicolumn{2}{c|}{\underset{\sim}{0}} & \frac{a_y^2T^3}{2} & a_y^2T^2 & \multicolumn{2}{c}{\underset{\sim}{0}} \\ \hline & & & & \frac{a_z^2T^4}{4} & \frac{a_z^2T^3}{2} \\ \multicolumn{2}{c|}{\underset{\sim}{0}} & \multicolumn{2}{c|}{\underset{\sim}{0}} & \frac{a_z^2T^3}{2} & a_z^2T^2 \end{array}\right] \tag{3.10}$$

The values assumed for a_x, a_y, and a_z for off-line calculation of the filter gains have a significant effect on the filter's performance. Much of the simulation work reported in Chapter IV is devoted to the study of the effects of different assumed values of a_x, a_y, and a_z on the filter's performance.

The Kalman filter requires that an apriori estimate of the system state vector be made. The initial state estimate vector used in all simulation runs is $\underset{\sim}{x}(0/-1) = \underset{\sim}{0}$, based on the reasoning that in a fixed cartesian coordinate system this is the mean of all possible initial state vectors. Since the system being simulated assumes a rotated cartesian coordinate system as described in Chapter II, it might be argued that the x coordinate position could be estimated as some anticipated detection range, and the x-coordinate velocity as a typical aircraft speed in the negative x-direction. However, since the filter converges rapidly as it is presently initialized, it is felt that to revise the initialization procedure would only serve to complicate the model and would provide no foreseeable benefit.

What is believed to be the simplest approach is taken again in choosing the initial covariance of estimation error matrix. It is desired that the initial position measurements be used as the initial position state estimates in order to have the filter track as quickly as possible. This end is accomplished quite simply by making the diagonal elements of the initial covariance of estimation error matrix

very large (e.g. 10^9). The effect of doing this can be seen by examining the Kalman filter gain equation

$$\underset{\sim}{G}(k) = \underset{\sim}{P}(k/k-1)\underset{\sim}{H}^T[\underset{\sim}{H}\underset{\sim}{P}(k/k-1)\underset{\sim}{H}^T+\underset{\sim}{R}(k)]^{-1} \qquad (3.11)$$

and considering the scalar case

$$G(k) = P(k/k-1)H[HP(k/k-1)H+R(k)]^{-1}$$

$$= \frac{P(k/k-1)H}{H^2P(k/k-1)+R(k)} \qquad (3.12)$$

where since $\underset{\sim}{H}$ is the observation matrix the scalar equivalent would simply be unity. If P(k/k-1) is made very large with respect to R(k) for k=0, then we have

$$G(0) = \frac{P(0/-1)}{P(0/-1)+R(0)} \simeq 1 \qquad (3.13)$$

which makes the first estimate equal the first measurement -- a logical result in the presence of a high degree of uncertainty.

The initial covariance of estimation error matrix used for all simulations is

$$\underset{\sim}{P}(0/-1) = \begin{bmatrix} 10^9 & & & \underset{\sim}{0} \\ & 10^9 & & \\ & & \ddots & \\ \underset{\sim}{0} & & & 10^9 \end{bmatrix} \qquad (3.14)$$

B. THE CORRELATED RANDOM ACCELERATION MODEL

One approach used to simulate the effects of target motion involves introducing correlated random acceleration inputs to a $1/S^2$ model. The correlated random accelerations are generated by an input of white noise into a coloring filter having a transfer function of $\alpha/(S+\alpha)$. The value of α can be selected to vary the time constant of the maneuver. This results in an overall transfer function for the model of target motion of

$$T(S) = \frac{\alpha}{S^2(S+\alpha)} \tag{3.15}$$

The 9×1 system state vector assumed for this model of target motion is

$$\underset{\sim}{x} = \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \ddot{x}(t) \\ y(t) \\ \dot{y}(t) \\ \ddot{y}(t) \\ z(t) \\ \dot{z}(t) \\ \ddot{z}(t) \end{bmatrix} \tag{3.16}$$

McKinley [4] describes this model in detail and gives the state difference equations (for one dimension) from which the state transition matrix

$$\underset{\sim}{\Phi}_i(T) = \begin{bmatrix} 1 & T & \frac{T}{\alpha} - \frac{1}{\alpha^2}\left[1-e^{-\alpha T}\right] \\ 0 & 1 & \frac{1}{\alpha}\left[1-e^{-\alpha T}\right] \\ 0 & 0 & e^{-\alpha T} \end{bmatrix} \tag{3.17}$$

and

$$\underset{\sim}{\Gamma}_i(T) = \begin{bmatrix} \frac{T^2}{2} - \frac{T}{\alpha} + \frac{1}{\alpha^2}\left[1-e^{-\alpha T}\right] \\ T - \frac{1}{\alpha}\left[1-e^{-\alpha T}\right] \\ 1 - e^{-\alpha T} \end{bmatrix} \tag{3.18}$$

can be extracted.

The matrices $\underset{\sim}{\Phi}_i(T)$ and $\underset{\sim}{\Gamma}_i(T)$ apply to a three-state system in one coordinate direction. For the full, nine-state three-coordinate system, the 9×9 matrix $\underset{\sim}{\Phi}(T)$ is

$$\underset{\sim}{\Phi}(T) = \begin{bmatrix} \underset{\sim}{\Phi}_x(T) & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{\Phi}_y(T) & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{\Phi}_z(T) \end{bmatrix} \tag{3.19}$$

where $\underset{\sim}{\Phi}_x(T)$, $\underset{\sim}{\Phi}_y(T)$, and $\underset{\sim}{\Phi}_z(T)$ are each equal to the 3×3 matrix $\underset{\sim}{\Phi}_i(T)$, and the 9×3 matrix $\underset{\sim}{\Gamma}(T)$ is

$$\underset{\sim}{\Gamma}(T) = \begin{bmatrix} \underset{\sim}{\Gamma}_x(T) & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{\Gamma}_y(T) & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{\Gamma}_z(T) \end{bmatrix} \tag{3.20}$$

where $\underset{\sim}{\Gamma}_x(T)$, $\underset{\sim}{\Gamma}_y(T)$ and $\underset{\sim}{\Gamma}_z(T)$ are each equal to the 3×1 vector $\underset{\sim}{\Gamma}_i(T)$.

In order to make the derivation of the $\underset{\sim}{Q}$ matrix less difficult to follow the following notation is adopted.

$$\underset{\sim}{\Gamma}(T) = \begin{bmatrix} \gamma_1 & 0 & 0 \\ \gamma_2 & 0 & 0 \\ \gamma_3 & 0 & 0 \\ 0 & \gamma_1 & 0 \\ 0 & \gamma_2 & 0 \\ 0 & \gamma_3 & 0 \\ 0 & 0 & \gamma_1 \\ 0 & 0 & \gamma_2 \\ 0 & 0 & \gamma_3 \end{bmatrix} \tag{3.21}$$

where

$$\gamma_1 = \frac{T^2}{2} - \frac{T}{\alpha} + \frac{1}{\alpha^2}\left[1-e^{-\alpha T}\right] , \tag{3.22}$$

$$\gamma_2 = T - \frac{1}{\alpha}\left[1-e^{-\alpha T}\right] , \text{ and} \tag{3.23}$$

$$\gamma_3 = 1 - e^{-\alpha T} . \tag{3.24}$$

The forcing input vector $\underset{\sim}{w}$ and the state excitation covariance matrix $\underset{\sim}{Q}$ are defined as in equations (3.8) and (3.9), respectively. If the matrix multiplication is done and the expectation operation performed on the product, the result is

$$\underset{\sim}{Q} = \begin{bmatrix} a_x^2 \underset{\sim}{q} & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & a_y^2 \underset{\sim}{q} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & a_z^2 \underset{\sim}{q} \end{bmatrix} \tag{3.25}$$

where

$$\underset{\sim}{q} = \begin{bmatrix} \gamma_1^2 & \gamma_1\gamma_2 & \gamma_1\gamma_3 \\ \gamma_1\gamma_2 & \gamma_2^2 & \gamma_2\gamma_3 \\ \gamma_1\gamma_3 & \gamma_2\gamma_3 & \gamma_3^2 \end{bmatrix} \tag{3.26}$$

The assumed covariance of measurement noise matrix, $\underset{\sim}{R}$, is the same as given in equation (3.7) for the constant-velocity system.

The constant α is inversely proportional to the assumed maneuver time constant. For example, the value of $\alpha=0.5$ corresponds to a time constant of two seconds which is considered to be typical for an aircraft evasive maneuver. Different time constants could conceivably be applied to each coordinate direction, however, in this thesis one value of α is applied throughout the system model.

The $\alpha/S^2(S+\alpha)$ model was simulated using values of $\alpha=0.1$, 0.3, 0.5 and 1.0. Based on the results of the simulations -- reported in Chapter IV -- $\alpha=0.5$ was chosen as the value providing the best overall performance in the limited number of tests conducted. This does not suggest that $\alpha=0.5$ is an optimum choice.

IV. MONTE-CARLO SIMULATION

The Monte-Carlo simulation program described in Appendix A was used to evaluate variations of the two basic models described in Chapter III.

A. TEST TRAJECTORIES

A set of four trajectories was chosen as a standard for use in simulating all models in order to facilitate comparison of the simulation results. All of the trajectories are products of the track generating program described in Appendix C. Each track is comprised of 60 state trajectory data samples with a sampling rate of four hertz.

The tracks are developed so that the initial target bearing coincides with the x axis to simulate the rotated coordinate system. The addition of measurement noise will cause the initial bearing to not coincide exactly with the x axis, however, no significant error is expected.

1. The Short-Range, Constant-Velocity Trajectory

The short-range, constant-velocity trajectory is a straight, inbound trajectory, beginning at a range of 3,850 yards at 30 degrees elevation and proceeding toward the origin with a range rate of 230 yd/s to a final range of 460 yards.

2. The Long-Range, Constant-Velocity Trajectory

The long-range, constant-velocity trajectory is a straight inbound track beginning at a range of 9,225 yards

at 30 degrees elevation and proceeding toward the origin with a range rate of 230 yd/s to a final range of 5,817 yards.

3. The Short-Range Trajectory With Maneuver

The short-range trajectory with a maneuver, like the short-range, constant-velocity track, begins at a range of 3,850 yards at 30 degrees elevation and proceeds toward the origin with a target speed of 230 yd/s. Commencing at a range of 3,500 yards the target undergoes a two-g, $\sin^2$ maneuver of 13.5 seconds duration. The maneuver is completed at the final sample range of 470 yards.

4. The Long-Range Trajectory With Maneuver

The long-range trajectory with a maneuver begins at a range of 9,225 yards at 30 degrees elevation. The target proceeds toward the origin with a speed of 230 yd/s. Commencing at a range of 8,900 yards the target undergoes a two-g, $\sin^2$ maneuver of 13.5 seconds duration. The maneuver is completed at the final sample range of 5,853 yards.

5. Other Trajectories

Representative variations of the two basic models were simulated using various trajectories in addition to the four standard trajectories described above in order to insure that the characteristics shown in the simulations were valid for trajectories other than the standard set.

The additional trajectories used include a constant-velocity crossing trajectory which has its initial point on the x-axis (simulating the rotated coordinate system) at a

range of 3,500 yards and proceeds to diverge from the x-axis at an angle of ten degrees as range decreases, and inbound trajectories with two-g and 2.5-g $\sin^2$ maneuvers of ten seconds duration occurring at various ranges.

B. ACCELERATIONS ASSUMED

The variations of the two basic system models differ primarily in the values of accelerations assumed for the components of the forcing input vector, $\underset{\sim}{w}(k)$, which is used in calculating the state excitation covariance matrix $\underset{\sim}{Q}(k)$. As indicated in equation (3.10) for the constant-velocity model, and equation (3.25) for the correlated random acceleration model, larger assumed values of acceleration result in larger corresponding entries in the $\underset{\sim}{Q}$ matrix. Solution of equations (1.7) through (1.9) shows that a consequence of larger $\underset{\sim}{Q}$ matrix entries is larger entries in the gain matrix $\underset{\sim}{G}(k)$. This results in the filter being more responsive to target maneuvers, but it is also more susceptible to estimation error caused by measurement noise. The assumption of different values for the components of $\underset{\sim}{w}(k)$ is done in an effort to determine the $\underset{\sim}{Q}$ matrix which best serves to compensate for modeling inadequacies as discussed in Chapter III.

The values of acceleration assumed are chosen as being representative of manned aircraft accelerations in the rotated cartesian coordinate system. It is predicted that an attacking aircraft during evasive maneuvering would have accelerations in the y and z-directions of three g's or less

and accelerations in the x-direction of less than one g. The values used for the simulation runs are listed in Tables II through IV along with the corresponding simulation prediction accuracy statistics.

C. SIMULATION RESULTS

The main criterion chosen for evaluation of model performance is the position-state prediction accuracy as discussed in Appendix B, although the means and covariances of estimation error are also calculated. The predictions for each model are calculated using the state transition matrix for the model as developed in Chapter III.

The statistics tabulated for comparison are the percentage of predictions per run which were in error by less than 24 feet -- labeled "hit %" -- and the percentage of predictions per run in error by less than 48 feet -- labeled "scare %".

The results of the various constant-velocity model simulations are listed in Table II. All statistics listed are products of 100 member Monte-Carlo ensembles.

The correlated-random-acceleration model was tested for several values of α as well as various assumed accelerations. The simulation results for $\alpha=0.3$ and $\alpha=0.5$ are listed in Tables III and IV respectively. All other values of α tested provided inferior performance and the results are not tabulated.

The simulation results listed in Tables II through IV are considered to be representative of all prediction

TABLE II CONSTANT-VELOCITY MODEL SIMULATION RESULTS

ASSUMED ACCELERATIONS (g)	CONSTANT-VELOCITY TRAJECTORIES				MANEUVERING TRAJECTORIES			
	Short-Range		Long-Range		Short-Range		Long-Range	
	Hit%	Scare %	Hit %	Scare %	Hit %	Scare %	Hit %	Scare %
$a_x=0$ $a_y=0$ $a_z=0$	86.4	91.4	34.7	59.4	5.9	9.14	0.0	0.095
$a_x=0.1$ $a_y=0.1$ $a_z=0.1$	86.4	91.4	32.4	59.2	5.9	11.5	0.0	0.095
$a_x=0.1$ $a_y=0.2$ $a_z=0.2$	88.4	91.4	25.2	58.3	13.0	27.4	0.0	0.67
$a_x=0.1$ $a_y=1.0$ $a_z=1.0$	81.4	91.2	4.62	21.57	15.2	28.5	0.33	1.10
$a_x=0.1$ $a_y=2.0$ $a_z=2.0$	74.2	90.8	2.0	9.1	13.8	31.3	0.14	0.86
$a_x=0.1$ $a_y=3.0$ $a_z=3.0$	63.4	86.1	1.1	6.1	13.8	32.4	0.29	0.95

TABLE III CORRELATED-RANDOM-ACCELERATION MODEL WITH $\alpha=0.3$

ASSUMED ACCELERATIONS (g)	CONSTANT-VELOCITY TRAJECTORIES				MANEUVERING TRAJECTORIES			
	Short-Range		Long-Range		Short-Range		Long-Range	
	Hit %	Scare %	Hit %	Scare %	Hit %	Scare %	Hit %	Scare %
$a_x=0.1$ $a_y=0.1$ $a_z=0.1$	71.3	79.1	2.9	15.1	5.0	18.6	0.0	0.4
$a_x=0.1$ $a_y=0.2$ $a_z=0.2$	71.3	79.1	3.0	15.1	13.8	22.6	0.1	0.3
$a_x=0.1$ $a_y=1.0$ $a_z=1.0$	70.6	78.9	1.5	8.3	8.5	28.7	0.1	0.4
$a_x=0.1$ $a_y=2.0$ $a_z=2.0$	66.7	78.6	0.8	3.9	8.6	20.1	0.0	0.1
$a_x=0.1$ $a_y=3.0$ $a_z=3.0$	62.9	77.7	0.4	2.1	6.4	21.7	0.0	0.1

TABLE IV CORRELATED-RANDOM-ACCELERATION MODEL WITH α=0.5

ASSUMED ACCELERATIONS (g)	CONSTANT-VELOCITY TRAJECTORIES				MANEUVERING TRAJECTORIES			
	Short-Range		Long-Range		Short-Range		Long-Range	
	Hit %	Scare %	Hit %	Scare %	Hit %	Scare %	Hit %	Scare %
a_x=0.1 a_y=0.5 a_z=0.5	73.4	81.1	6.6	24.0	4.6	18.8	not tested	
a_x=0.1 a_y=1.0 a_z=1.0	73.4	81.1	6.0	23.8	21.2	30.2	not tested	
a_x=0.2 a_y=1.0 a_z=1.0	72.8	81.1	4.6	20.5	20.4	30.2	not tested	
a_x=0.1 a_y=2.0 a_z=2.0	73.1	81.1	4.1	18.7	17.3	29.3	not tested	
a_x=0.2 a_y=2.0 a_z=2.0	72.3	81.1	3.2	16.7	16.5	29.1	not tested	

accuracy statistics obtained, with the exception that Table II does not present statistics indicating the effect on filter performance when the assumed x-direction acceleration a_x, is varied. It was determined early in the simulation phase that the constant-velocity model performs best against the test trajectories used when a_x is approximately 0.1 g, therefore in the interest of computer time conservation only this value of a_x was used for the final set of simulation runs.

Also in order to conserve computer time the correlated-random-acceleration model with α=0.5 was not tested against the long-range trajectory with a maneuver. This is considered a justifiable omission since no appreciable improvement or further degradation of prediction accuracy in this area is anticipated.

In Table V the five best performing filters against each test trajectory are ranked according to the prediction accuracy statistics shown in Table II through IV. The "hit" and "scare" percentages for each ranked filter are included with the model description. For the long-range, maneuvering target only two of the estimators tested produced "scare" percentages of one percent -- the filter ranked highest is chosen because it produced smaller average miss distances than the second and third ranked filters for 17 of 21 predictions per run even though it scored only two "scares" in the 100-member ensemble.

TABLE V FILTER PERFORMANCE RANKINGS

SHORT-RANGE CONSTANT-VELOCITY TRAJECTORY	LONG-RANGE CONSTANT-VELOCITY TRAJECTORY	SHORT-RANGE MANEUVERING TRAJECTORY	LONG-RANGE MANEUVERING TRAJECTORY
(1)$1/S^2$ Model a_x=0.1g $a_y=a_z$=0.2g (88.4/91.4)	(1)$1/S^2$ Model $a_x=a_y=a_z$=0 (34.7/59.2)	(1)$0.5/S^2(S+0.5)$Model a_x=0.1g $a_y=a_z$=1.0g (21.2/30/2)	(1)$1/S^2$ Model $a_x=a_y=a_z$=0.1g (0.0/0.1) Lower Average Miss Distance for 17 of 21 predictions per run
(2)$1/S^2$ Model $a_x=a_y=a_z$=0.1g or $a_x=a_y=a_z$=0.0 (86.4/92.4)	(2)$1/S^2$ Model $a_x=a_y=a_z$=0.1g (32.4/59.2)	(2)$0.5/S^2(S+0.5)$Model a_x=0.2g $a_y=a_z$=1.0g (20.4/32.2)	(2)$1/S^2$ Model a_x=0.1g $a_y=a_z$=1.0g (0.3/1.1)
(3)$1/S^2$ Model a_x=0.1g $a_y=a_z$=1.0g (81.4/91.2)	(3)$1/S^2$ Model a_x=0.1g $a_y=a_z$=0.2g (24.2/58.3)	(3)$0.5/S^2(S+0.5)$Model a_x=0.1g $a_z=a_z$=2.0g (17.3/29.3)	(3)$1/S^2$ Model a_x=0.1g $a_y=a_z$=3.0g (0.1/1.0)
(4)$1/S^2$ Model a_x=0.1g $a_y=a_z$=2.0g (74.2/90.8)	(4)$0.5/S^2(S+0.5)$Model a_x=0.1g $a_y=a_z$=0.5g (6.6/24.0)	(4)$1/S^2$ Model a_x=0.1g $a_y=a_z$=3.0g (13.8/32.4)	
(5)$0.5/S^2(S+0.5)$Model a_x=0.1g $a_y=a_z$=1.0g (73.4/81.1)	(5)$0.5/S^2(S+0.5)$Model a_x=0.1g $a_y=a_z$=1.0g (6.0/23.8)	(5)$1/S^2$ Model a_x=0.1g $a_y=a_z$=2.0g (13.8/31.3)	

D. COMPARISON OF PREDICTION ACCURACY AND ESTIMATION ERROR STATISTICS

An effort to correlate the prediction accuracy statistics with the estimation error means and variances for the sample estimate from which the prediction is made indicates that for constant-velocity targets, where the estimation error means are small for all the models, the model with the smaller variances of estimation error -- especially for the velocity state estimates -- can be expected to produce more consistently accurate predictions. For example, a comparison of the statistics for two $1/S^2$ models, one with forcing inputs assumed to be zero and the other with assumed values of 0.1 g in the x-direction and two g's in the y and z-directions, shows that both filters in general have small means of estimation error. The estimate at sample number 18, $\hat{\underset{\sim}{x}}(18/18)$, upon which the prediction of target position at the time of sample number 53, $\hat{\underset{\sim}{x}}(53/18)$, is based in the long-range, constant-velocity trajectory, has small means of estimation error for both filters (less than one yard for position states and one yd/s for velocity states). The variances of y and z-direction velocity estimation error for the model which assumes two-g acceleration in the y and z directions are much larger (an order of magnitude larger for the y and z-direction velocity states). This model scores 18 "scares" at sample number 53 over the 100 run ensemble while the model which assumes zero forcing inputs scores 100

"scares". The trend indicated in this example holds for all constant-velocity trajectory statistics tested.

A similar analysis of the statistics generated for short-range maneuvering targets, where the means of estimation error are in some cases more than 20 yards for the position states and more than 30 yd/s for the velocity states, indicates that the better prediction accuracy statistics are produced by the model with the lower means of estimation error. The variance of estimation error does not appear to be a significant factor at short range in any of the models tested. Notice that the mean estimation errors are much larger than the measurement errors, this is due to the target maneuver occurring after the constant-velocity filter has reached steady state.

At long ranges, any "hits" scored against a maneuvering target are considered to be either purely by chance or due to the type of maneuver which the target executes (i.e. the target may pass close to the predicted position even though it has maneuvered radically during the projectile time-of-flight).

The average prediction miss distance is considered to be the best prediction accuracy statistic for use in evaluating a filter's performance against a long-range, maneuvering target. The model ranked first against long-range maneuvering targets in Table V has larger means and smaller variances of estimation error than the second and third ranked models. These statistics indicate that the estimator is not

tracking the target accurately, but is generating predictions of future target position in the right general area. This is suspected to be largely because the target maneuvers about an axis of symmetry in the test trajectory rather than at random.

V. ADAPTIVE FILTERS

A. THE SWITCH-ON-RANGE ADAPTIVE FILTER

The simpler of the two adaptive filtering methods proposed is the switch-on-range adaptive filter. This scheme is based on the idea that since the measurement noise in cartesian coordinates is proportional to the target range, relatively high filter gains can be used on a short-range target without significant degradation of the estimate quality for a constant-velocity target and with significant improvement of the estimate quality for a maneuvering target.

Upon target detection the estimator uses a gain schedule which is based on a small forcing inputs -- such as 0.1 g in each coordinate direction. This "low Q" gain schedule is continued until the target range is less than a predetermined threshold. Upon reaching the switching threshold range the estimator switches to a steady-state gain matrix which is based on relatively large forcing inputs ("high Q").

The $1/S^2$ model is used for both the "low Q" and "high Q" filters. This model was chosen because of its superior performance against constant-velocity targets, especially at long range, as indicated in the Monte-Carlo simulation results of Chapter IV.

The "high Q" filter uses a steady-state gain matrix because it is assumed that a good estimate is available at the time the switching occurs.

The switching threshold range for simulation purposes is chosen as 3,333 yards. The simulation results of Chapter IV indicate that the "high Q" filters based on the $1/S^2$ model with assumed accelerations of one to three g's in the y and z-directions perform relatively well within this range against either maneuvering or constant-velocity targets.

The "high Q" gain matrix used in the simulation is the steady-state gain matrix of the $1/S^2$ model with assumed accelerations of 0.1 g in the x-direction and three g's in the y and z-directions. The gains are computed from the model developed in Chapter III.

A FORTRAN listing of the switch-on-range adaptive filter subroutine is included in Appendix D.

B. SWITCH-ON-RESIDUALS ADAPTIVE FILTER #1

The switch-on-residuals adaptive filter is more sophisticated and more complex than the switch-on-range filter previously described.

The first version of the switch-on-residuals adaptive filter is designed to track with a "low Q" gain schedule based on the constant-velocity model until a component of the filter residual vector

$$\underset{\sim}{\rho}(k) = \underset{\sim}{Z}(k)-\underset{\sim\sim}{H}\hat{\underset{\sim}{x}}(k/k-1) \tag{5.1}$$

exceeds a calculated gating level,

$$\ell_i(k) = 2\sqrt{P_{ii}(k/k)+R_{ii}(k)} \quad , \; i=1,2,3, \tag{5.2}$$

on two successive samples.

$P_{ii}(k/k)$ is the theoretical variance of position estimate error and $R_{ii}(k)$ is the theoretical variance of measurement noise in one coordinate direction.

The values used for the theoretical variance of position estimate error are taken from the steady-state theoretical covariance of estimation error matrix for the model from which the "high Q" gains are calculated.

The values used for the theoretical variance of measurement noise in the gating level calculation are calculated on-line from the diagonal terms of the covariance of measurement noise matrix given in equation (2.14) and vary with the target location.

If the occurrence of a residual value greater than the gating level at time i is defined as event A_i and the probability of this occurring on the next sample is event A_{i+1}, A_i and A_{i+1} are independent events. The probability of two successive samples with residuals greater than the gating level is

$$P(A_iA_{i+1}) = P(A_i)P(A_{i+1}) \ . \qquad (5.3)$$

If only the $R_{ii}(k)$ term of the equation (5.2) is considered, then the gating level is equal to twice the standard deviation of measurement noise. For a normal distribution the probability of an event differing from the mean by more than two standard deviations is given as

$$P(A_i) = P(A_{i+1}) = 0.0228 \ .$$

Substitution of this value into equation (5.3) yields a probability of

$$P(A_i A_{i+1}) = (0.0228)^2 = 5.20 \times 10^{-4}$$

which is the probability that the residual is greater than the gating level due to estimation error or measurement noise when no target maneuver is experienced.

The filter is independently adaptive in each coordinate direction. For example, when $\rho_1(k)$ is greater than $\ell_1(k)$ for two successive samples, the x-direction states are re-initialized by accepting the latest measurement as the position estimate and beginning at k=0 with a "high Q" gain schedule based on the $\alpha/S^2(S+\alpha)$ system model with values of and assumed forcing inputs chosen to provide good performance against a maneuvering target.

The y-direction and z-direction state estimates continue to be based on the constant-velocity ($1/S^2$) model. The $\alpha/S^2(S+\alpha)$ filter gains continue to be used in making the x-direction estimates until three successive samples have residuals less than one-half the gating level.

Then the inequality

$$\rho_1(k) < \sqrt{P_{11}(k/k)+R_{11}(k)} \qquad (5.4)$$

is satisfied for three successive samples the filter switches back to the "low Q" gain schedule assuming the same time index as would have been used for the current sample if no adaptive switching had occurred.

For the Monte-Carlo simulation of the switch-on-residuals filter, the "low Q" gains are based on the $1/S^2$ model with assumed forcing inputs of 0.1 g in each coordinate direction since it is proposed that those gains will be used exclusively against constant-velocity targets. This choice is based on the simulation results listed in Chapter IV.

Also based on the simulation results listed in Chapter IV, the model chosen for calculation of the "high Q" gains is the $\alpha/S^2(S+\alpha)$ model with $\alpha=0.5$ and assumed forcing inputs of 0.1 g in the x-direction and one g in the y and z-directions.

C. SWITCH-ON-RESIDUALS ADAPTIVE FILTER #2

The second version of the switch-on-residuals adaptive filter differs from the first in the manner in which the switching gates are calculated.

In this version the gating level is calculated as

$$\ell_i(k) = 3\sqrt{P_{ii}(k/k)+R_{ii}(k)} \quad , \quad i=1,2,3 \qquad (5.5)$$

and the switch is made to the "high Q" gain schedule when the gating level is exceeded by one sample residual.

The switch back to the "low Q" gain schedule occurs when the residual is less than 0.88 $\ell_i(k)$ for two successive samples (the coefficient 0.88 is an arbitrary choice for a value greater than 0.5 but less than one). In this version when the filter switches back to the "low Q" gains the index for the "high Q" gain schedule is not reset immediately but is delayed one sample so that if the residual exceeds the

gating level on the first sample after the filter returns to the "low Q" gain schedule, the filter does not reinitialize by accepting the position measurement as the estimate but simply switches back to the "high Q" gain schedule using the same time index as would have been used if the switch to the "low Q" gain schedule had not taken place.

The gain schedules used in this filter are the same as those used in the first version of the switch-on-residuals adaptive filter described in the previous section.

D. THE COMPROMISE FILTER

The compromise filter is an attempt to pick one gain schedule which provides adequate performance for all types of trajectories without adaptive switching. The filter simulation results listed in Chapter IV indicate no clear choice for the compromise filter.

Because of its good performance against constant-velocity targets at all ranges and its acceptable performance against short-range maneuvering targets, the $1/S^2$ model which assumes accelerations of 0.1 g in the x-direction and 0.2 g in the y and z-directions is chosen for calculation of the compromise filter gains.

E. SUMMARY OF ADAPTIVE FILTER DEVELOPMENT

Four adaptive filters have been developed for evaluation by Monte-Carlo simulation, Table VI provides a summary of their key features.

TABLE VI SUMMARY OF ADAPTIVE FILTERS

ADAPTIVE FILTER	LOW Q FILTER MODEL	HIGH Q FILTER MODEL	SWITCH TO HIGH Q FILTER	SWITCH TO LOW Q FILTER
Switch-on-Range	$1/S^2$ Model $a_x=a_y=a_z=0.1g$	$1/S^2$ Model $a_x=0.1g$ $a_y=a_z=3g$	Range < 3333 yd	Range $\geq$ 3333 yd
Switch-on Residuals #1	$1/S^2$ Model $a_x=a_y=a_z=0.1g$	$\alpha/S^2(S+\alpha)$Model $\alpha=0.5$ $a_x=0.1g$ $a_y=a_z=1.0g$	$\rho_i>2\sqrt{P_{ii}(k/k)+R_{ii}(k)}$ for 2 successive samples	$\rho_i<1/2\sqrt{P_{ii}(k/k)+R_{ii}(k)}$ for 3 successive samples
Switch-on Residuals #2	$1/S^2$ Model $a_x=a_y=a_z=0.1g$	$\alpha/S^2(S+\alpha)$Model $\alpha=0.5$ $a_x=0.1g$ $a_y=a_z=1.0g$	$\rho_i>3\sqrt{P_{ii}(k/k)+R_{ii}(k)}$ for one sample	$\rho_i<0.88\sqrt{P_{ii}(k/k)+R_{ii}(k)}$ for 2 successive samples

VI. ADAPTIVE FILTER SIMULATION

The Monte-Carlo simulation program described in Appendix A is used in conjunction with the adaptive filter subroutines listed in Appendix D to evaluate the performance of the three adaptive filter schemes and the compromise non-adaptive filter proposed in Chapter V.

A. TEST TRAJECTORIES

A set of six test trajectories was used for the simulation of each of the adaptive filter configurations and the compromise filter. Each test trajectory is a product of the track generating program described in Appendix C and is comprised of 160 state trajectory data samples with a four-hertz sampling rate. All six of the test trajectories represent a target proceeding inbound from a range of 8,000 yards and an elevation of 20 degrees at a speed of 200 yd/s with a dive angle of 20 degrees. The trajectories differ in the target maneuvers incorporated:

Trajectory No. 1 - No maneuver

Trajectory No. 2 - A two-g maneuver of 13.5 seconds duration commencing at a range of 4,000 yards.

Trajectory No. 3 - A two-g maneuver of 13.5 seconds duration commencing at a range of 2,700 yards.

Trajectory No. 4 - A two-g maneuver of 13.5 seconds duration commencing at a range of 3,333 yards.

Trajectory No. 5 - A 2.5-g maneuver of 10 seconds duration commencing at a range of 2,700 yards.

Trajectory No. 6 - A 2.5-g maneuver of 10 seconds duration commencing at a range of 4,000 yards.

B. SIMULATION RESULTS

The prediction accuracy statistics produced by the Monte-Carlo simulation of the switch-on-range and switch-on-residuals adaptive filters and the compromise constant-velocity filter are listed in Table VII. The statistics presented are products of 80 member ensembles.

The adaptive filter predictions are generated using the constant-velocity model state transition matrix for all configurations. A small number of simulation runs made using the correlated-random-acceleration model state transition matrix produced slightly lower "hit" and "scare" percentages.

TABLE VII MONTE-CARLO SIMULATION RESULTS FOR ADAPTIVE FILTERS

TEST TRAJECTORY NO.	SWITCH-ON-RANGE		#1 SWITCH-ON-RESIDUALS		#2 SWITCH-ON-RESIDUALS		COMPROMISE CONSTANT-VELOCITY	
	Hit %	Scare %	Hit %	Scare %	Hit %	Scare %	Hit %	Scare %
1	81.8	93.4	74.4	81.4	86.6	92.6	85.7	93.8
2	47.2	57.8	33.8	45.4	44.2	60.0	40.4	56.9
3	60.1	75.3	47.9	60.2	58.3	72.0	55.2	69.8
4	52.3	65.7	39.7	53.2	50.4	63.3	45.1	65.1
5	60.2	75.2	45.5	60.7	59.6	73.0	56.7	70.8
6	54.7	67.8	41.8	53.8	51.9	65.9	45.5	64.2

VII. SUMMARY AND CONCLUSIONS

A. TEST RESULT SUMMARY

1. Simulation Result Summary for the Two Basic Models

The simulation results presented in Chapter IV show that the constant-velocity ($1/S^2$) model with relatively small values of assumed forcing inputs provides the better performance against constant-velocity targets at both short range and long range. By increasing the value assumed for the forcing inputs to the $1/S^2$ model its performance against maneuvering targets can be improved but there is a corresponding loss in the performance for constant-velocity targets, particularly at long range where the transverse measurement noise components are relatively large.

The correlated-random-acceleration model provides better performance against maneuvering targets and reasonably good performance against constant-velocity targets at short range, but its performance against long-range, constant-velocity targets, as measured by the prediction accuracy statistics listed in Chapter IV, is inferior to the "low Q" $1/S^2$ models.

None of the models tested provided good prediction accuracy against the long-range maneuvering target. This is to be expected because of the long time of flight over which the prediction must be made. Even a perfect estimator would surely provide inaccurate predictions when the predictions must be made ten seconds ahead on a target which maneuvers

in the interim. Average miss distance was chosen as the most appropriate statistic for evaluating a filter's performance against a long-range, maneuvering target.

2. Adaptive Filter Simulation Summary

The simulation results for the adaptive filters and the compromise filter listed in Chapter VI indicate that the switch-on-range adaptive filter provides the best overall performance against the six targets trajectories included in the test. Although the performance of the second version of the switch-on-residuals adaptive filter is comparable.

It was anticipated that the more sophisticated switch-on-residuals adaptive filter would provide the better performance, however, even the second version of the switch-on-residuals filter, which was partially developed by "cut and try" modifications to the original version, does not better the performance of the simpler, switch-on-range adaptive filter. The problem encountered with switch-on-residual filter seems to be that measurement noise causes the filter to switch to the "high Q" gain schedule when tracking long range constant-velocity targets.

The compromise filter, the simplest approach, provides fair performance -- better than the original version of the switch-on-residuals adaptive filter -- and serves as a standard for comparison with the adaptive filter performance.

B. SUGGESTIONS FOR FURTHER INVESTIGATION

One of the most obvious extensions of the simulation work is to incorporate the rotated coordinate system algorithm developed in Chapter II into the Monte-Carlo program for complete testing.

The switch-on-residuals adaptive filter should receive more attention. This is believed to be potentially the better of the adaptive filtering schemes but the most effective combination of gating levels and switching rules has not been found.

The switch-on-range adaptive filter should be simulated using other values for the steady state "high Q" gain matrix. A prime candidate is the steady-state gain matrix of the correlated-random-acceleration model with $\alpha=0.5$ and assumed forcing inputs of 0.1 g in the x-direction and one g in the y and z directions.

The various models should also be simulated using other types of trajectories including a proportional navigation trajectory representing an anti-ship missile attack.

More effort should be given to developing a definite quantitative correlation between prediction accuracy and estimation error statistics as discussed briefly in Chapter IV. A thorough investigation of relationships between the two sets of statistics may provide useful information for use in filter design.

C. CONCLUSIONS

The rotated cartesian coordinate system has been shown to have potential advantages over the fixed cartesian coordinate system both in diminished correlation of measurement noise and in selection of the components of assumed forcing inputs for the system model.

The prediction accuracy statistics are concise and valid measures of the system's performance in most cases, however the mean and covariance of estimation error are valuable statistics to have available to support the prediction accuracy statistics and to serve as measures of filter performance in cases such as the long range maneuvering target, where the prediction accuracy measurement provides little information.

The constant-velocity model with small values of assumed forcing inputs has been shown to provide the best performance against constant-velocity aircraft while the correlated-random-acceleration model with the proper choice of α to match the maneuver time constant provides the best performance against maneuvering targets.

Finally, the relatively simple switch-on-range adaptive filter and the more complex switch-on-residuals adaptive filter have been shown to provide improved performance over the compromise non-adaptive filter.

APPENDIX A

THE MONTE-CARLO SIMULATION PROGRAM

A.1 Program Description

The general purpose Monte-Carlo simulation program developed by Prof. D. E. Kirk of the Naval Postgraduate School is quite flexible in its basic form and offers several options to the user. The various options and many of the input data requirements are explained by comments early in the program listing.

A.2 The Monte-Carlo Simulation Program Listing

```
C      THIS PROGRAM PERFORMS MONTE CARLO SIMULATION OF STATE ESTIMATORS
C      OF WHICH THE KALMAN FILTER IS ONE EXAMPLE. THERE ARE SEVERAL
C      OPTIONS AVAILABLE AS INDICATED IN THE DETAILED COMMENTS BELOW..
C      IT SHOULD BE NOTED THAT ALL COMPUTATIONS OF GAINS USING THE
C      SUBROUTINE GAIN ARE PERFORMED IN DOUBLE PRECISION. THUS ALL
C      ARRAYS FOR USE IN "GAIN" MUST BE PREPARED ACCORDINGLY.
C
       REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI
       COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,
      19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9
      2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),
      3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9
      4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),
      5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,IIRO,IXZ,IV,IW,IEST,ND,NFIRE,NHIT
      6,NSCARE,ICOORD,IADAPT
       DIMENSION XP(160), YP(160), KHIT(160)
C
CC
CCC    N=ORDER OF SYSTEM MODEL AND FILTER (DIMENSION OF X,XHAT)
CCC
CCC    M=NUMBER OF MEASUREMENTS (DIMENSION OF THE VECTOR Z)
CCC
CCC    IN=NUMBER OF INPUT RANDOM FORCING FCNS (=DIMENSION OF W)
CCC
CCC    NSAM=NUMBER OF TIME SAMPLES
CCC
CCC    NENS=NUMBER OF MEMBERS IN ENSEMBLE
C
       READ (5,1) N,M,IN,NSAM,NENS
     1 FORMAT (5(I10))
C
CCC
CCC    THE VALUE OF ND READ IN MUST EQUAL THE ROW (AND COLUMN) DIMENSION
CCC    SPECIFIED FOR THE SQUARE MATRIX "TEMP1", E.G. IF TEMP1(3,3) IS
C      SPECIFIED IN THE COMMON STATEMENT "ND" MUST BE EQUAL TO 3.
       READ (5,2) ND
C
CCC    THE FLAG IADAPT CAUSES THE FILTER TO BE ADAPTIVE IF  IT IS NOT
CCC    ZERO. SUBROUTINE ADAPT MUST BE PROVIDED BY THE USER IF THIS OPTION
CCC    IS DESIRED.
C
       READ (5,2) IADAPT
     2 FORMAT (I2)
C
CCC
CCC    IG=-1 -- GAINS COMPUTED OFF-LINE AND READ IN
C          0 -- GAINS COMPUTED ONLY ONCE BEFORE STARTING MONTE CARLO
C          1 -- GAINS COMPUTED FOR EACH MEMBER OF ENSEMBLE
```

```
C
C     IFLR=0 -- R IS READ IN                                                    490
C     IFLR.NE.0 -- R IS COMPUTED ON-LINE AT EACH TIME SAMPLE                    500
C                                                                               510
C     IEST=0 -- STANDARD KALMAN FILTER EQUATION IS USED                         520
C     IEST.NE.0 -- STD. KALMAN FILTER EQ. NOT USED                              530
C                                                                               540
C     ITRK= 0 -- SEVERAL TRACKS GENERATED FROM STD LINEAR EQS                   550
C     ITRK= 1 -- ONLY ONE TRACK IS USED                                         560
C      ITRK.NE.0OR1 -- SEVERAL TRACKS GENERATED BUT NOT FROM STD.               570
C                      LINEAR DIFFERENCE EQUATIONS                              580
C                                                                               590
C     ISTAT=0 -- MEAN OF TRACK,MEAN & VARIANCE OF EST. ERROR COMPUTED           600
C     ISTAT.NE.0 -- SAME AS ISTAT=0 BUT OFF-DIAGONAL TERMS IN COVARIANCE        610
C                                                                               620
C                   MATRIX ARE ALSO COMPUTED                                    630
C                                                                               640
      READ (5,3) IG,IFLR,IEST,ITRK,ISTAT,IQ,ITRO,IMISS                          650
    3 FORMAT (8(I10))                                                           660
C                                                                               670
C     IG=-1 -- Q IS COMPUTED ON-LINE AT EACH SAMPLE BY "QON"                    680
C         0 -- Q MATRIX IS READ IN                                              690
C         1 -- COVARIANCE OF W READ IN AND Q IS  COMPUTED ONCE BEFORE           700
C              MONTE CARLO BEGINS BY CALLING "QMAT"                             710
C                                                                               720
C     ITRO=0 -- ONLY USED IF ONE TRACK; ITRO=0 INDICATES THE                    730
C               TRACK IS READ IN (S.P.), OTHERWISE THE TRACK                    740
C               IS GENERATED BY A SINGLE CALL OF SUBROUTINE                     750
C               TRACK.                                                          760
C                                                                               770
C                                                                               780
C     IMISS= 0---PROJECTILE MISS DISTANCE NOT COMPUTED. OTHERWISE IT IS         790
C              CALCULATED BY CALL OF SUBROUTINE BURST.                          800
C                                                                               810
C     IPRT=0 -- SOME OR ALL OUTPUT DATA IS PRINTED                              820
C                                                                               830
C     IPLT=0 -- SOME OR ALL OUTPUT DATA IS PLOTTED                              840
      READ (5,4) IPRT,IPLT                                                      850
    4 FORMAT (2(I5))                                                            860
C                                                                               870
C     IGPLT,ITHVPL,IMTPLT,ISMPLT,ISVPLT ARE PLOTTING INDICATORS                 880
C     IF THE INDICATOR IS EQUAL TO 1 PLOTS ARE GENERATED. THE INDICATORS        890
C     CORRESPOND TO THE FOLLOWING QUANTITIES                                    900
C     IGPLT(GAIN VS K),ITHVPL (THEORETICAL COVARIANCE MATRIX VS K),             910
C     IMTPLT (SAMPLE MEAN OF TRACK=TRACK IF ONLY ONE TRACK),                    920
C     ISMPLT (SAMPLE MEAN OF EST. ERROR VS K), ISVPLT (SAMPLE VARIANCE          930
C     OF EST. ERROR VS K)                                                       940
      IF (IPLT.EQ.0) READ (5,5) IGPLT,ITHVPL,IMTPLT,ISMPLT,ISVPLT               950
                                                                                960
```

```
    5 FORMAT (5(I10))                                                       970
C                                                                           980
C     IPHI=0 -- PHI MATRIX TO BE READ IN (IN DOUBLE PREC)                   990
C     IPHI.NE.0 -- PHI MATRIX NOT TO BE READ                               1000
C                                                                          1010
C     IH=0 -- H MATRIX TO BE READ IN (IN D.P.)                             1020
C                                                                          1030
C     IR=0 -- R MATRIX IS READ IN                                          1040
C                                                                          1050
C     IPKKM1=0 -- THE STARTING VALUE FOR THE MATRIX P(K/K-1) IS TO         1060
C                 BE READ IN (D.P.)                                        1070
C                                                                          1080
C     IGAM=0 -- GAMMA MATRIX TO BE READ IN (D.P.)                          1090
C                                                                          1100
C     ISIGV=0 -- STD. DEVIATIONS OF MEASUREMENT NOISE TO BE READ IN (SP)   1110
C                                                                          1120
C     ISIGW=0 -- STD. DEVIATIONS OF RANDOM INPUTS TO BE READ IN (SP)       1130
C                SIGW MUST BE READ IN IF ITRK = 0.                         1140
C                                                                          1150
C     IXHZ=0 -- INITIAL VALUE OF THE PREDICTED VALUE XH(0/-1) TO           1160
C               BE READ IN (S.P.)                                          1170
C                                                                          1180
C     IC=1 -- ONE INITIAL CONDITION VALUE USED FOR THE STATE               1190
C             OTHERWISE THE VALUE OF X(0) IS GENERATED BY A RANDOM         1200
C             NIMBER GENERATOR CALLED BY "XZERO"                           1210
C                                                                          1220
C                                                                          1230
      READ (5,6) IPHI,IH,IR,IPKKM1,IGAM,ISIGV,ISIGW,IXHZ,IC               1240
    6 FORMAT (10I5)                                                        1250
      CALL OVFLOW                                                          1260
      IH = 6395217                                                         1270
      IV = 1936748                                                         1280
      IXZ = 135769                                                         1290
C                                                                          1300
C                                                                          1310
C     THE FOLLOWING SECTION PRINTS OUT A DESCRIPTION OF THE RUN AS         1320
C     SPECIFIED BY THE USER'S FLAGS                                        1330
C                                                                          1340
C                                                                          1350
      WRITE (6,143)                                                        1360
      WRITE (6,7)                                                          1370
    7 FORMAT (20X,'DESCRIPTION OF RUN',//)                                 1380
      IF (IG.EQ.0) GO TO 9                                                 1390
      IF (IG.EQ.1) GO TO 11                                                1400
      WRITE (6,8)                                                          1410
    8 FORMAT (10X,'GAINS COMPUTED OFF-LINE AND READ IN',/)                 1420
      GO TO 13                                                             1430
    9 WRITE (6,10)                                                         1440
```

```
10 FORMAT (10X,'GAINS COMPUTED ONCE IN "GAIN" BEFORE STARTING MONTE C    1450
     1ARLO',/)                                                          1460
       GO TO 13                                                         1470
11   WRITE (6,12)                                                       1480
12   FORMAT (10X,'GAINS COMPUTED FOR EACH MEMBER OF ENSEMBLE',/)        1490
13   IF (IEST.EQ.0) GO TO 15                                            1500
       WRITE (6,14)                                                     1510
14   FORMAT (10X,'THE STANDARD LINEAR EQS. DO NOT CHARACTERIZE THE FILT 1520
     1ER',/)                                                            1530
       GO TO 17                                                         1540
15   WRITE (6,16)                                                       1550
16   FORMAT (10X,'THE STD. KALMAN EQS. CHARACTERIZE THE LINEAR FILTER', 1560
     1/)                                                                1570
17   IF (ITRK.EQ.0) GO TO 23                                            1580
       IF (ITRK.EQ.-1) GO TO 21                                         1590
       IF (ITRO.EQ.0) GO TO 19                                          1600
       WRITE (6,18)                                                     1610
18   FORMAT (10X,'ONLY ONE TRACK IS USED AND IT IS GENERATED BY SUBROUT 1620
     1INE TRACK',/)                                                     1630
       GO TO 25                                                         1640
19   WRITE (6,20)                                                       1650
20   FORMAT (10X,'ONLY ONE TRACK IS USED AND IT IS READ IN',/)          1660
       GO TO 25                                                         1670
21   WRITE (6,22)                                                       1680
22   FORMAT (10X,'SEVERAL TRACKS USED BUT NOT GENERATED FROM STD. LINEA 1690
     1R DIFFERENCE EQS.',/)                                             1700
       GO TO 25                                                         1710
23   WRITE (6,24)                                                       1720
24   FORMAT (10X,'SEVERAL TRACKS GENERATED BY USING THE STD. LINEAR DIF 1730
     1FERENCE EQS.',/)                                                  1740
25   IF (ISTAT.EQ.0) GO TO 27                                           1750
       WRITE (6,26)                                                     1760
26   FORMAT (10X,'MEAN OF TRACK, MEAN OF EST. ERROR AND COVARIANCE OF E 1770
     1ST. ERROR ARE COMPUTED',/)                                        1780
       GO TO 29                                                         1790
27   WRITE (6,28)                                                       1800
28   FORMAT (10X,'MEAN OF TRACK, MEAN AND VARIANCES OF EST. ERROR ARE   1810
     1COMPUTED',/)                                                      1820
29   IF (IQ.EQ.0) GO TO 33                                              1830
       IF (IQ.EQ.1) GO TO 31                                            1840
       WRITE (6,30)                                                     1850
30   FORMAT (10X,'THE Q MATRIX IS COMPUTED ON-LINE AT EACH SAMPLE BY "Q 1860
     1ON"',/)                                                           1870
       GO TO 35                                                         1880
31   WRITE (6,32)                                                       1890
32   FORMAT (10X,'THE COVARIANCE OF W IS READ IN AND Q IS COMPUTED BY "  1900
     1QMAT" BEFORE STARTING MONTE CARLO',/)                             1910
       GO TO 35                                                         1920
```

```
   33 WRITE (6,34)                                                      1930
   34 FORMAT (10X,'THE Q MATRIX IS READ IN',/)                          1940
   35 IF (IFLR.EQ.0) GO TO 37                                           1950
      WRITE (6,36)                                                      1960
   36 FORMAT (10X,'R IS COMPUTED ON-LINE AT EACH SAMPLE BY "RON"',/)    1970
      GO TO 39                                                          1980
   37 WRITE (6,38)                                                      1990
   38 FORMAT (10X,'R IS READ IN',/)                                     2000
   39 WRITE (6,40)                                                      2010
   40 FORMAT (////,20X,'INPUT DATA CALLED FOR',//)                      2020
      IF (IPHI.EQ.0) WRITE (6,41)                                       2030
   41 FORMAT (10X,'PHI MATRIX',/)                                       2040
      IF (IH.EQ.0) WRITE (6,42)                                         2050
   42 FORMAT (10X,'H MATRIX',/)                                         2060
      IF (IFLR.EQ.0) WRITE (6,43)                                       2070
   43 FORMAT (10X,'R MATRIX',/)                                         2080
      IF (IQ.EQ.0) GO TO 45                                             2090
      IF (IQ.EQ.-1) GO TO 47                                            2100
      WRITE (6,44)                                                      2110
   44 FORMAT (10X,'COVARIANCE OF W',/)                                  2120
      GO TO 47                                                          2130
   45 WRITE (6,46)                                                      2140
   46 FORMAT (10X,'Q MATRIX',/)                                         2150
   47 IF (IGAM.EQ.0) WRITE (6,48)                                       2160
   48 FORMAT (10X,'GAMMA MATRIX',/)                                     2170
      IF (ISIGV.EQ.0) WRITE (6,49)                                      2180
   49 FORMAT (10X,'STANDARD DEVIATIONS OF MEASUREMENT NOISE',/)         2190
      IF (ISIGW.EQ.0) WRITE (6,50)                                      2200
   50 FORMAT (10X,'STANDARD DEVIATIONS OF INPUT FORCING W',/)           2210
      IF (IXHZ.EQ.0) WRITE (6,51)                                       2220
   51 FORMAT (10X,'XHAT(0/-1)',/)                                       2230
      IF (IC.NE.1) WRITE (6,52)                                         2240
   52 FORMAT (10X,'MEANS AND VARIANCES OF X(0)',/)                      2250
      IF (IPKKM1.EQ.0) WRITE (6,53)                                     2260
   53 FORMAT (10X,'P(0/-1)',/)                                          2270
C                                                                       2280
C                                                                       2290
C     THE FOLLOWING SECTION PRINTS OUT A DESCRIPTION OF THE OUTPUT      2300
C     DATA CALLED FOR                                                   2310
C                                                                       2320
C                                                                       2330
      WRITE (6,54)                                                      2340
   54 FORMAT (////,20X,'OUTPUT CALLED FOR',//)                          2350
      IF (IPRT.NE.0) GO TO 60                                           2360
      WRITE (6,55)                                                      2370
   55 FORMAT (10X,'PRINTED OUTPUT OF THE FOLLOWING DATA',/)             2380
      IF (IG.EQ.0) WRITE (6,56)                                         2390
   56 FORMAT (15X,'GAIN MATRICES AND THEORETICAL COVARIANCE OF EST. ERRO 2400
```

```
     1R MATRICES',/)                                                                    2410
       WRITE (6,57)                                                                     2420
    57 FORMAT (15X,'SAMPLE MEANS OF TRACK AND ESTIMATION ERROR,SAMPLE VAR               2430
     1IANCES OF EST. ERROR',/)                                                          2440
       IF (ISTAT.NE.0) WRITE (6,58)                                                     2450
    58 FORMAT (15X,'COVARIANCE OF ESTIMATION ERROR MATRIX',/)                           2460
       IF (IMISS.NE.0) WRITE (6,59)                                                     2470
    59 FORMAT (15X,'MEAN PROJECTILE MISS DISTANCE FOR EACH SAMPLE ',/)                  2480
       GO TO 62                                                                         2490
    60 WRITE (6,61)                                                                     2500
    61 FORMAT (10X,'NO PRINTED OUTPUT CALLED FOR',//)                                   2510
    62 IF (IPLT.NE.0) GO TO 69                                                          2520
       WRITE (6,63)                                                                     2530
    63 FORMAT (10X,'THE FOLLOWING PLOTS ARE CALLED FOR',/)                              2540
       IF (IGPLT.EQ.1) WRITE (6,64)                                                     2550
    64 FORMAT (15X,'G(K) VS. K',/)                                                      2560
       IF (ITHVPL.EQ.1) WRITE (6,65)                                                    2570
    65 FORMAT (15X,'P(K/K) THEORETICAL VS. K',/)                                        2580
       IF (IMTPLT.EQ.1) WRITE (6,66)                                                    2590
    66 FORMAT (15X,'MEAN OF TRACK VS. K',/)                                             2600
       IF (ISMPLT.EQ.1) WRITE (6,67)                                                    2610
    67 FORMAT (15X,'SAMPLE MEANS OF ESTIMATION ERROR VS. K',/)                          2620
       IF (ISVPLT.EQ.1) WRITE (6,68)                                                    2630
    68 FORMAT (15X,'SAMPLE VARIANCES OF ESTIMATION ERROR VS. K',/)                      2640
       GO TO 71                                                                         2650
    69 WRITE (6,70)                                                                     2660
    70 FORMAT (10X,'NO PLOTS CALLED FOR',/)                                             2670
    71 CONTINUE                                                                         2680
       WRITE (6,143)                                                                    2690
       WRITE (6,72)                                                                     2700
    72 FORMAT (20X,'INPUT DATA',///)                                                    2710
       WRITE (6,73) N,M,IN,NSAM,NENS                                                    2720
    73 FORMAT (4X,'N=',I2,4X,'M=',I2,4X'IN=',I2,4X'NSAM=',I3,4X,'NENS=',I               2730
     15,//)                                                                             2740
C                                                                                       2750
CC                                                                                      2760
CC     THE FOLLOWING SECTION READS THE SPECIFIED INPUT MATRICES                         2770
C                                                                                       2780
       IF (IPHI.NE.0) GO TO 76                                                          2790
       CALL MREAD (PHI,N,N)                                                             2800
C                                                                                       2810
       DO 74 I=1,N                                                                      2820
C                                                                                       2830
       DO 74 J=1,N                                                                      2840
    74 PHIS(I,J) = PHI(I,J)                                                             2850
C                                                                                       2860
       WRITE (6,75)                                                                     2870
    75 FORMAT (//,10X,'THE PHI MATRIX IS',/)                                            2880
```

```
      CALL MWRITE (PHI,N,N)                                              2890
C                                                                        2900
C                                                                        2910
   76 IF (IH.NE.0) GO TO 79                                              2920
      CALL MREAD (H,M,N)                                                 2930
C                                                                        2940
      DO 77 I=1,M                                                        2950
C                                                                        2960
      DO 77 J=1,N                                                        2970
   77 HS(I,J) = H(I,J)                                                   2980
C                                                                        2990
      WRITE (6,78)                                                       3000
   78 FORMAT (//,10X,'THE H MATRIX IS',/)                                3010
      CALL MWRITE (H,M,N)                                                3020
C                                                                        3030
C                                                                        3040
   79 IF (IR.NE.0) GO TO 81                                              3050
      CALL MREAD (R,M,M)                                                 3060
      WRITE (6,80)                                                       3070
   80 FORMAT (//,10X,'THE R MATRIX IS',/)                                3080
      CALL MWRITE (R,M,M)                                                3090
C                                                                        3100
C                                                                        3110
   81 IF (IQ.NE.1) GO TO 83                                              3120
      CALL MREAD (COVW,IN,IN)                                            3130
      WRITE (6,82)                                                       3140
   82 FORMAT (//,10X,'THE COVARIANCE OF W MATRIX IS',/)                  3150
      CALL MWRITE (COVW,IN,IN)                                           3160
      GO TO 85                                                           3170
   83 IF (IQ.NE.0) GO TO 85                                              3180
      CALL MREAD (Q,N,N)                                                 3190
      WRITE (6,84)                                                       3200
   84 FORMAT (//,10X,'THE Q MATRIX IS',/)                                3210
      CALL MWRITE (Q,N,N)                                                3220
C                                                                        3230
C                                                                        3240
   85 IF (IGAM.NE.0) GO TO 88                                            3250
      CALL MREAD (GAMMA,N,IN)                                            3260
C                                                                        3270
      DO 86 I=1,N                                                        3280
C                                                                        3290
      DO 86 J=1,IN                                                       3300
   86 GAMMAS(I,J) = GAMMA(I,J)                                           3310
C                                                                        3320
      WRITE (6,87)                                                       3330
   87 FORMAT (//,10X,'THE GAMMA MATRIX IS',/)                            3340
      CALL MWRITE (GAMMA,N,IN)                                           3350
C                                                                        3360
```

```
C                                                                       3370
   88 IF (IPKKM1.NE.0) GO TO 90                                         3380
      CALL MREAD (PKKM1,N,N)                                            3390
      WRITE (6,89)                                                      3400
   89 FORMAT (//,10X,'THE MATRIX P(0/-1) IS',/)                         3410
      CALL MWRITE (PKKM1,N,N)                                           3420
C                                                                       3430
C                                                                       3440
   90 IF (ISIGV.NE.0) GO TO 92                                          3450
      CALL VREAD (SIGV,M)                                               3460
      WRITE (6,91)                                                      3470
   91 FORMAT (//,10X,'THE STD. DEVIATIONS OF MEASUREMENT NOISE ARE',/)  3480
      CALL VWRITE (SIGV,M)                                              3490
C                                                                       3500
C                                                                       3510
   92 IF (ISIGW.NE.0) GO TO 94                                          3520
      CALL VREAD (SIGW,IN)                                              3530
      WRITE (6,93)                                                      3540
   93 FORMAT (//,10X,'THE STD. DEVIATIONS OF INPUT FORCING W ARE',/)    3550
      CALL VWRITE (SIGW,IN)                                             3560
C                                                                       3570
C                                                                       3580
   94 IF (IXHZ.NE.0) GO TO 96                                           3590
      CALL VREAD (XHATZ,N)                                              3600
      WRITE (6,95)                                                      3610
   95 FORMAT (//,10X,'THE VECTOR XHAT(0/-1) IS',/)                      3620
      CALL VWRITE (XHATZ,N)                                             3630
C                                                                       3640
C                                                                       3650
   96 IF (IC.EQ.1) GO TO 99                                             3660
C     IC.NE.1 MEANS THAT MEANS AND STD. DEVIATIONS OF THE INITIAL STATE 3670
C     VALUE MUST BE READ IN. OTHERWISE NOT READ IN.                     3680
      CALL VREAD (XZMEAN,N)                                             3690
      WRITE (6,97)                                                      3700
   97 FORMAT (//,10X,'THE MEAN OF THE VECTOR X(0) IS',/)                3710
      CALL VWRITE (XZMEAN,N)                                            3720
C                                                                       3730
C                                                                       3740
      CALL VREAD (SIGXZ,N)                                              3750
      WRITE (6,98)                                                      3760
   98 FORMAT (//,10X,'THE STANDARD DEVIATIONS OF THE VECTOR X(0) ARE',/)3770
      CALL VWRITE (SIGXZ,N)                                             3780
      GO TO 107                                                         3790
C                                                                       3800
C                                                                       3810
   99 READ (5,102) (XS(I,1),I=1,N)                                      3820
C     INITIAL CONDITION HAS BEEN READ                                   3830
      WRITE (6,100)                                                     3840
```

```
  100 FORMAT (//,10X,'THE INITIAL STATE IS',/)                                  3850
      WRITE (6,106) (XS(I,1),I=1,N)                                             3860
      IF (ITRK.NE.1) GO TO 107                                                  3870
      IF (ITRC.NE.0) GO TO 103                                                  3880
C                                                                               3890
      DO 101 K=2,NSAM                                                           3900
  101 READ (5,102) (XS(I,K),I=1,N)                                              3910
C                                                                               3920
  102 FORMAT (4F20.12)                                                          3930
      GO TO 104                                                                 3940
  103 CALL TRACK                                                                3950
C     IF TRACK CALLED HERE IT SHOULD BE WRITTEN TO GENERATE AND                 3960
C     STORE THE TRACK IN XS(N,K) FOR K=2(NOTE),NSAM                             3970
C                                                                               3980
  104 WRITE (6,105)                                                             3990
  105 FORMAT (//,10X,'THE FIRST AND LAST POINTS ON THE SINGLE TRACK TO B        4000
     1E USED ARE',/)                                                            4010
      WRITE (6,106) (XS(I,1),I=1,N)                                             4020
      WRITE (6,106) (XS(I,NSAM),I=1,N)                                          4030
  106 FORMAT (9(2X,1PE12.5),/)                                                  4040
  107 CONTINUE                                                                  4050
C                                                                               4060
CCCC                                                                            4070
CCCC  THE FOLLOWING SECTION PREPARES FOR THE MONTE CARLO LOOP                   4080
CCCC                                                                            4090
CCCC  FORM NXN IDENTITY MATRIX IN DOUBLE PRECISION                              4100
C                                                                               4110
      DO 108 I=1,N                                                              4120
C                                                                               4130
      DO 108 J=1,N                                                              4140
      EI(I,J) = 0.D0                                                            4150
  108 IF (I.EQ.J) EI(I,J)=1.D0                                                  4160
C                                                                               4170
CC    GIVEN THE MATRIX GAMMA AND THE COVARIANCE OF W COMPUTE Q                  4180
C     USING DOUBLE PRECISION ARITHMETIC                                         4190
      IF (IQ.NE.1) GO TO 109                                                    4200
      CALL QMAT                                                                 4210
      WRITE (6,84)                                                              4220
      CALL MWRITE (Q,N,N)                                                       4230
  109 IF (IG.NE.-1) GO TO 112                                                   4240
C                                                                               4250
      DO 110 K=1,40                                                             4260
C                                                                               4270
      DO 110 I=1,N                                                              4280
  110 READ (5,102) (GKS(I,J,K),J=1,M)                                           4290
C                                                                               4300
C                                                                               4310
      DO 111 K=41,NSAM                                                          4320
```

```
C                                                                       4330
      DO 111 I=1,N                                                      4340
C                                                                       4350
      DO 111 J=1,M                                                      4360
  111 GKS(I,J,K) = GKS(I,J,40)                                          4370
C                                                                       4380
      GO TO 115                                                         4390
  112 IF (IG.NE.0) GO TO 115                                            4400
C                                                                       4410
      DO 114 K=1,NSAM                                                   4420
      CALL GAIN                                                         4430
C                                                                       4440
      DO 114 I=1,N                                                      4450
C                                                                       4460
      DO 113 L=1,N                                                      4470
  113 PKKS(I,L,K) = PKK(I,L)                                            4480
C                                                                       4490
C                                                                       4500
      DO 114 J=1,M                                                      4510
  114 GKS(I,J,K) = G(I,J)                                               4520
C                                                                       4530
  115 CONTINUE                                                          4540
C     IF GAINS WERE TO BE READ IN (IG=-1) OR COMPUTED ONLY              4550
C     ONCE(IG=0), THIS HAS NOW BEEN DONE                                4560
CCC                                                                     4570
CCC                                                                     4580
C     SET UP ARRAYS FOR COMPUTING STATISTICS                            4590
      NFIRE = 0                                                         4600
      NHIT = 0                                                          4610
      NSCARE = 0                                                        4620
C                                                                       4630
      DO 116 K=1,NSAM                                                   4640
      AMISS(K) = 0.0                                                    4650
      KHIT(K) = 0                                                       4660
C                                                                       4670
      DO 116 J=1,N                                                      4680
      XM(J,K) = 0.                                                      4690
      ERR(J,K) = 0.                                                     4700
C                                                                       4710
      DO 116 L=1,N                                                      4720
  116 VAR(J,L,K) = 0.                                                   4730
C                                                                       4740
CCC                                                                     4750
CCC                                                                     4760
CCC   BEGIN MAIN ITERATION LOOP HERE                                    4770
C                                                                       4780
      DO 123 ITER=1,NENS                                                4790
      IF (IC.EQ.1) GO TO 117                                            4800
```

```
      CALL XZERO                                                        4810
C                                                                       4820
  117 DO 118 I=1,N                                                      4830
  118 XHKKM1(I) = XHATZ(I)                                              4840
C                                                                       4850
C                                                                       4860
      DO 123 K=1,NSAM                                                   4870
C     FORM NOISY MEASUREMENT FROM TRUE STATE VALUE                      4880
C                                                                       4890
C                                                                       4900
      DO 119 I=1,N                                                      4910
  119 X(I) = XS(I,K)                                                    4920
C                                                                       4930
      CALL MEAS                                                         4940
C     GAIN IS NOT TO BE COMPUTED ON-LINE IF IG.NE.1                     4950
      IF (IG.NE.1) GO TO 121                                            4960
C                                                                       4970
C     Q IS TO BE COMPUTED ON-LINE IF IFLQ.NE.0                          4980
      IF (IFLQ.NE.0) CALL QON                                           4990
C     R IS TO BE COMPUTED ON-LINE IF IFLR.NE.0                          5000
C                                                                       5010
      IF (IFLR.NE.0) CALL RON                                           5020
      CALL GAIN                                                         5030
C                                                                       5040
      DO 120 I=1,N                                                      5050
C                                                                       5060
      DO 120 J=1,M                                                      5070
  120 GKS(I,J,K) = G(I,J)                                               5080
C                                                                       5090
C                                                                       5100
C     UPDATE THE STATE ESTIMATE                                         5110
C                                                                       5120
  121 CALL ESTIM                                                        5130
C                                                                       5140
C     UPDATE RUNNING SUMS USED IN COMPUTING STATISTICS                  5150
      CALL STAT                                                         5160
      IF (IMISS.EQ.0) GO TO 122                                         5170
      CALL BURST (KHIT)                                                 5180
  122 IF (K.EQ.NSAM) GO TO 123                                          5190
C                                                                       5200
C     UPDATE TRACK BY COMPUTING X(K+1)                                  5210
      IF (ITRK.NE.1) CALL TRACK                                         5220
  123 CONTINUE                                                          5230
C                                                                       5240
C                                                                       5250
C     DIVIDE RUNNING SUMS COMPUTED BY SUBROUTINE STAT BY ENSEMBLE       5260
C     SIZE TO COMPUTE STATISTICS                                        5270
      ENS = NENS                                                        5280
```

```
C                                                                       5290
      DO 125 K=1,NSAM                                                   5300
C                                                                       5310
      DO 125 J=1,N                                                      5320
      IF (ITRK.EQ.1) GO TO 124                                          5330
      XM(J,K) = XM(J,K)/ENS                                             5340
  124 ERR(J,K) = ERR(J,K)/ENS                                           5350
  125 VAR(J,J,K) = VAR(J,J,K)/ENS-ERR(J,K)**2                           5360
C                                                                       5370
      IF (ISTAT.EQ.0) GO TO 127                                         5380
C                                                                       5390
C     COMPUTE OFF-DIAGONAL TERMS IN COVARIANCE OF ESTIMATION            5400
C     ERROR MATRIX IF ISTAT.NE.0                                        5410
C                                                                       5420
      DO 126 K=1,NSAM                                                   5430
C                                                                       5440
      DO 126 L=2,N                                                      5450
      LM1 = L-1                                                         5460
C                                                                       5470
      DO 126 J=1,LM1                                                    5480
  126 VAR(L,J,K) = VAR(L,J,K)/ENS-ERR(L,K)*ERR(J,K)                     5490
C                                                                       5500
  127 CONTINUE                                                          5510
C                                                                       5520
      IF (IMISS.EQ.0) GO TO 129                                         5530
C     DIVIDE RUNNING SUM OF MISS DISTANCES COMPUTED BY SUBROUTINE       5540
C     BURST BY ENSEMBLE SIZE TO COMPUTE AVERAGE MISS DISTANCES.         5550
C                                                                       5560
      DO 128 K=1,NSAM                                                   5570
      AMISS(K) = AMISS(K)/ENS                                           5580
      FIRE = FLOAT(NFIRE)/ENS                                           5590
      HIT = FLOAT(NHIT)/ENS                                             5600
  128 SCARE = FLOAT(NSCARE)/ENS                                         5610
C                                                                       5620
C                                                                       5630
  129 IF (IPRT.NE.0) GO TO 151                                          5640
      WRITE (6,130)                                                     5650
  130 FORMAT ('1',20X,'OUTPUT DATA',///)                                5660
C     WRITE GAINS, THEORETICAL COVARIANCES OF ESTIMATION ERROR          5670
C     IF ONE SET OF GAINS HAS BEEN USED.                                5680
      IF (IADAPT.NE.0) GO TO 137                                        5690
      IF (IG.NE.0) GO TO 137                                            5700
      WRITE (6,131)                                                     5710
  131 FORMAT (10X,'THE GAIN MATRICES ARE',/)                            5720
C                                                                       5730
      DO 133 K=1,NSAM                                                   5740
      WRITE (6,132) K                                                   5750
  132 FORMAT (5X,'K=',I3,/,10X,'G(K)=',/)                               5760
```

```
C                                                                       5770
      DO 133 I=1,N                                                      5780
  133 WRITE (6,106) (GKS(I,J,K),J=1,M)                                  5790
C                                                                       5800
      WRITE (6,134)                                                     5810
  134 FORMAT (1X,//,10X,'THE THEORETICAL COVARIANCE MATRIX IS',/)        5820
C                                                                       5830
      DO 136 K=1,NSAM                                                   5840
      WRITE (6,135) K                                                   5850
  135 FORMAT (5X,'K=',I3,/,10X,'P(K/K)=',/)                             5860
C                                                                       5870
      DO 136 I=1,N                                                      5880
  136 WRITE (6,106) (PKKS(I,J,K),J=1,N)                                 5890
C                                                                       5900
CC                                                                      5910
C                                                                       5920
  137 WRITE (6,143)                                                     5930
      WRITE (6,138)                                                     5940
      WRITE (6,139)                                                     5950
  138 FORMAT (T5,'TIME',T16,'VECTOR COM-',T34,'SAMPLE MEAN',            5960
     1 T51,'SAMPLE MEAN OF',T71,'SAMPLE VARIANCE OF')                   5970
  139 FORMAT (T5,'INDEX',T16,'PONENT INDEX',T34,'OF TRACK',             5980
     1 T51,'ESTIMATION ERROR',T71,'ESTIMATION ERROR')                   5990
  140 FORMAT (6X,I3,13X,I1,10X,1PE14.7,2(6X,1PE14.7))                   6000
  141 FORMAT (//)                                                       6010
C                                                                       6020
      DO 142 K=1,NSAM                                                   6030
      WRITE (6,141)                                                     6040
C                                                                       6050
      DO 142 I=1,N                                                      6060
  142 WRITE (6,140) K,I,XM(I,K),ERR(I,K),VAR(I,I,K)                     6070
C                                                                       6080
      WRITE (6,143)                                                     6090
      IF (ISTAT.EQ.0) GO TO 147                                         6100
  143 FORMAT ('1')                                                      6110
      WRITE (6,144)                                                     6120
  144 FORMAT (10X,'THE SAMPLE COVARIANCE OF EST. ERROR MATRIX IS',//)    6130
C                                                                       6140
      DO 145 K=1,NSAM                                                   6150
      WRITE (6,146) K                                                   6160
C                                                                       6170
      DO 145 I=1,N                                                      6180
  145 WRITE (6,106) (VAR(I,L,K),L=1,I)                                  6190
C                                                                       6200
  146 FORMAT (//,2X,'K=',I3,/)                                          6210
      WRITE (6,143)                                                     6220
  147 IF (IMISS.EQ.0) GO TO 151                                         6230
C                                                                       6240
```

```
      DO 148 K=1,NSAM                                                    6250
  148 WRITE (6,149) K,AMISS(K),K,KHIT(K)                                 6260
C                                                                        6270
  149 FORMAT (' ',T10,'AMISS(',I3,') = ',G20.12,10X,'KHIT(',I3,') = ',I4 6280
     1)                                                                  6290
      WRITE (6,150) FIRE,HIT,SCARE                                       6300
  150 FORMAT ('0',T10,'AVE. NO. FIRED = ',G12.4,5X,'AVE. NO. HITS = ',G1 6310
     12.4,5X,'AVE. NO. SCARES AND/OR HITS = ',G12.4)                     6320
      WRITE (6,143)                                                      6330
  151 IF (IPLT.NE.0) GO TO 172                                           6340
C                                                                        6350
      DO 152 K=1,NSAM                                                    6360
  152 XP(K) = K                                                          6370
C                                                                        6380
      IF (IGPLT.NE.1) GO TO 156                                          6390
C                                                                        6400
      DO 154 I=1,N                                                       6410
C                                                                        6420
      DO 154 J=1,M                                                       6430
C                                                                        6440
      DO 153 K=1,NSAM                                                    6450
  153 YP(K) = GKS(I,J,K)                                                 6460
C                                                                        6470
      WRITE (6,143)                                                      6480
      CALL PLOTP (XP,YP,NSAM,0)                                          6490
  154 WRITE (6,155) I,J                                                  6500
C                                                                        6510
  155 FORMAT (12X,'G(',I1,',',I1,')  VS. K')                             6520
  156 IF (ITHVPL.NE.1) GO TO 160                                         6530
C                                                                        6540
      DO 158 I=1,N                                                       6550
C                                                                        6560
      DO 157 K=1,NSAM                                                    6570
  157 YP(K) = PKKS(I,I,K)                                                6580
C                                                                        6590
      WRITE (6,143)                                                      6600
      CALL PLOTP (XP,YP,NSAM,0)                                          6610
  158 WRITE (6,159) I,I                                                  6620
C                                                                        6630
  159 FORMAT (12X,'PKK(',I1,',',I1,')  VS. K')                           6640
  160 IF (IMTPLT.NE.1) GO TO 164                                         6650
C                                                                        6660
      DO 162 I=1,N                                                       6670
C                                                                        6680
      DO 161 K=1,NSAM                                                    6690
  161 YP(K) = XM(I,K)                                                    6700
C                                                                        6710
      WRITE (6,143)                                                      6720
```

```
      CALL PLOTP (XP,YP,NSAM,0)                                          6730
  162 WRITE (6,163) I                                                    6740
C                                                                        6750
  163 FORMAT (12X,'MEAN OF X(',I1,')  VS. K')                            6760
  164 IF (ISMPLT.NE.1) GO TO 168                                         6770
C                                                                        6780
      DO 166 I=1,N                                                       6790
C                                                                        6800
      DO 165 K=1,NSAM                                                    6810
  165 YP(K) = ERR(I,K)                                                   6820
C                                                                        6830
      WRITE (6,143)                                                      6840
      CALL PLOTP (XP,YP,NSAM,0)                                          6850
  166 WRITE (6,167) I,I                                                  6860
C                                                                        6870
  167 FORMAT (12X,'XHATKK(',I1,') -X(',I1,')  VS. K')                    6880
  168 IF (ISVPLT.NE.1) GO TO 172                                         6890
C                                                                        6900
      DO 170 I=1,N                                                       6910
C                                                                        6920
      DO 169 K=1,NSAM                                                    6930
  169 YP(K) = VAR(I,I,K)                                                 6940
C                                                                        6950
      WRITE (6,143)                                                      6960
      CALL PLOTP (XP,YP,NSAM,0)                                          6970
  170 WRITE (6,171) I                                                    6980
C                                                                        6990
  171 FORMAT (12X,'ERROR VARIANCE(',I1,') VS.. K')                       7000
      WRITE (6,143)                                                      7010
  172 CONTINUE                                                           7020
      STOP                                                               7030
      END                                                                7040
C
CC
C
      SUBROUTINE ADAPT                                                     10
C
CC        THIS SUBROUTINE CONTROLS THE ADAPTIVE SWITCHING OF THE FILTER      20
CC        PARAMETERS IN ORDER TO MAKE THE FILTER MORE RESPONSIVE IN A        30
CC        CHANGING SITUATION.                                                40
CCC                                                                          50
C                                                                            60
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI            70
      COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,     80
     19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9     90
     2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),    100
     3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9    110
     4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),     120
```

```
     5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,IW,IEST,ND,NFIRE,NHIT      130
     6,NSCARE,ICOORD,IADAPT                                                   140
      DIMENSION GKA(9,9,41)                                                   150
C                                                                             160
CCC   THE USER MUST SUPLY STATEMENTS HERE TO CONTROL THE ADAPTIVE             170
CCCC  FILTER OPERATION.                                                       180
CCC                                                                           190
C                                                                            1030
   18 RETURN                                                                 1040
      END                                                                    1050
C
CCC
      SUBROUTINE BURST (KHIT)                                                  10
C                                                                              20
CCCCC    THIS SUBROUTINE COMPUTES PROJECTILE TIME OF FLIGHT AND OUTPUTS       30
CCCCC    A SET OF  STATISTICS INDICATING THE ACCURACY OF THE PREDICTED        40
CCCCC    STATE VALUES WHEN THE TARGET IS WITHIN GUN RANGE. AS WRITTEN         50
CCCCC    THIS SUBROUTINE IS VALID FOR CARTESIAN COORDINATES ONLY.             60
C                                                                              70
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI              80
      COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,       90
     19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9      100
     2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),      110
     3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9      120
     4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),       130
     5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,IW,IEST,ND,NFIRE,NHIT      140
     6,NSCARE,ICOORD,IADAPT                                                   150
      DIMENSION XEST(9,160), XF(9), TEMPP(9,9), KHIT(160)                     160
      RING1 = 8.0                                                             170
      RING2 = 16.0                                                            180
      SUMSQ = 0.0                                                             190
      ERRSQ = 0.0                                                             200
      TSAMP = 0.25                                                            210
      TOFCON = 1.4285E-03                                                     220
C                                                                             230
CCC   TOFCON REPRESENTS THE SLOPE OF A STRAIGHT LINE APPROXIMATION OF         240
CCC   THE TIME OF FLIGHT (TOF) VS RANGE CURVE FOR A 5 IN./54 CAL. GUN         250
CCC   (SEC/YARD)                                                              260
C                                                                             270
      N1 = N/3                                                                280
      N2 = 2*N1                                                               290
      NN = N-1                                                                300
C                                                                             310
      DO 1 I=1,N                                                              320
C                                                                             330
CC    STORE THE CURRENT ESTIMATE FOR LATER USE IN CALCULATING THE             340
C     PREDICTION.                                                             350
```

```
C                                                                       360
    1 XEST(I,K) = XHKK(I)                                               370
C                                                                       380
C                                                                       390
      DO 2 I=1,NN,N1                                                    400
C                                                                       410
C     CALCULATE THE RANGE ESTIMATE FROM CARTESIAN CORDINATE POSITION    420
C     STATE ESTIMATES.                                                  430
C                                                                       440
    2 SUMSQ = SUMSQ+XHKK(I)**2                                          450
C                                                                       460
      D = SQRT(SUMSQ)                                                   470
C                                                                       480
C     TIME OF FLIGHT IS DIRECTLY PROPORTIONAL TO RANGE                  490
C                                                                       500
      TOF = D*TOFCON                                                    510
C                                                                       520
C     CALCULATE THE NUMBER OF SAMPLES WHICH WILL BE TAKEN DURING THE    530
C     PROJECTILE TIME OF FLIGHT,AND STORE AS VARIABLE INTOF.            540
C                                                                       550
      INTOF = TOF/TSAMP                                                 560
      IF (INTOF.GE.K) GO TO 3                                           570
C                                                                       580
C     IF T. O. F. IS GREATER THAN 10 SEC. THE TARGET IS BEYOND THE      590
C     MAXIMUM EFFECTIVE FIRING RANGE .                                  600
C                                                                       610
      IF ((INTOF*TSAMP).LE.10.0) GO TO 4                                620
C                                                                       630
C     IF THE TARGET IS NOT FIRED UPON, SET IN A LARGE VALUE OF MISS     640
C     DISTANCE AS AN INDICATOR.                                         650
C                                                                       660
    3 AMISS(K) = AMISS(K)+9999.9                                        670
      RETURN                                                            680
C                                                                       690
C     COUNT THE NUMBER OF SHOTS FIRED PER RUN.                          700
C                                                                       710
    4 NFIRE = NFIRE+1                                                   720
C                                                                       730
      DO 5 I=1,N                                                        740
C                                                                       750
C     THE PREDICTION FOR THE GUNFIRE SOLUTION WAS MADE FROM AN ESTIMATE 760
C     WHICH WAS GENERATED TOF SECONDS EARLIER.                          770
C                                                                       780
    5 XF(I) = XEST(I,K-INTOF)                                           790
C                                                                       800
C                                                                       810
C     THE FOLLOWING ALGORITHM (THRU STATEMENT NO. 23) CALCULATES THE    820
C     STATE PREDICTION TRANSITION MATRIX, PHI**INTOF.                   830
```

```
C                                                                       840
      SUM1 = 0.0                                                        850
      SUM2 = 0.0                                                        860
      TOF = FLOAT(INTOF)                                                870
C                                                                       880
      DO 6 I=1,N                                                        890
C                                                                       900
      DO 6 J=1,N                                                        910
    6 TEMPP(I,J) = 0.0                                                  920
C                                                                       930
      TEMPP(1,1) = 1.0                                                  940
      TEMPP(1,2) = TSAMP*TOF                                            950
      TEMPP(2,2) = 1.0                                                  960
      IF (N.NE.9) GO TO 9                                               970
C                                                                       980
      DO 7 I=1,INTOF                                                    990
    7 SUM1 = SUM1+PHIS(3,3)**(I-1)                                      1000
C                                                                       1010
      JTOF = INTOF-1                                                    1020
C                                                                       1030
      DO 8 I=1,JTOF                                                     1040
    8 SUM2 = SUM2+(INTOF-I)*PHIS(3,3)**(I-1)                            1050
C                                                                       1060
      TEMPP(1,3) = PHIS(1,3)*SUM1+TSAMP*PHIS(2,3)*SUM2                  1070
      TEMPP(2,3) = PHIS(2,3)*SUM1                                       1080
      TEMPP(3,3) = PHIS(3,3)**INTOF                                     1090
    9 IF (N.LT.6) GO TO 11                                              1100
C                                                                       1110
      DO 10 NA=N1,N2,N1                                                 1120
C                                                                       1130
      DO 10 I=1,N1                                                      1140
      INA = I+NA                                                        1150
C                                                                       1160
      DO 10 J=1,N1                                                      1170
      JNA = J+NA                                                        1180
   10 TEMPP(INA,JNA) = TEMPP(I,J)                                       1190
C                                                                       1200
   11 CONTINUE                                                          1210
C                                                                       1220
C     XHAT(K/K-INTOF)=(PHI**INTOF)*XHAT(K-INTOF/K-INTOF)                1230
C                                                                       1240
      CALL VPROD (TEMPP,XF,N,N,VTMP)                                    1250
C                                                                       1260
      DO 12 I=1,NN,N1                                                   1270
C                                                                       1280
C     PREDICTION ERROR = X(K)-XHAT(K/K-INTOF)                           1290
C                                                                       1300
      ERD = XS(I,K)-VTMP(I)                                             1310
```

```
   12 ERRSQ = ERRSQ+ERD**2                                                  1320
C                                                                           1330
CCCC                                                                        1340
CCCC  MISS DISTANCE = SQRT(SUM OF SQUARED POSITION ERRORS)                  1350
C                                                                           1360
      BURST = SQRT(ERRSQ)                                                   1370
      AMISS(K) = AMISS(K)+BURST                                             1380
C                                                                           1390
CCCC  IF THE MISS DISTANCE IS LESS THAN THE RADIUS RING2 THE PREDICTION     1400
CCCC  IS CONSIDERED TO BE A NEAR MISS OR SCARE.                             1410
C                                                                           1420
      IF (BURST.GT.RING2) GO TO 13                                          1430
C                                                                           1440
CCC   COUNT THE NUMBER OF HITS AND THE NUMBER OF SCARES PER RUN.            1450
C                                                                           1460
      NSCARE = NSCARE+1                                                     1470
      KHIT(K) = KHIT(K)+1                                                   1480
C                                                                           1490
CCCCC COUNT THE NUMBER OF HITS AND/OR SCARES PER SAMPLE OVER THE            1500
CCCCC ENSEMBLE.                                                             1510
CCCCC                                                                       1520
CCCCC IF THE MISS DISTANCE IS LESS THAN THE RADIUS RING1 THE PREDICTION     1530
CCCCC IS CONSIDERED TO BE A HIT.                                            1540
C                                                                           1550
      IF (BURST.LE.RING1) NHIT=NHIT+1                                       1560
   13 RETURN                                                                1570
      END                                                                   1580
C
CCC
C
      SUBROUTINE ESTIM                                                        10
C
CCCCC THIS SUBROUTINE UPDATES THE STATE ESTIMATE. IN THE DEFAULT            20
CCCCC CONDITION (IEST.EQ.0) THE STANDARD EQUATIONS                          30
CCCCCC                                                                      40
CCCCCC XHAT(K/)=XHAT(K/K-1)+G(K)*(Z(K)-H(K)*XHAT(K/K-1))                    50
CCCCCC                                                                      60
CCCCCC XHAT(K+1/K)=PHI*XHAT(K/K)                                            70
CCCCCC                                                                      80
C     ARE EVALUATED                                                         90
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI          100
      COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,   110
     19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9  120
     2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),  130
     3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9  140
     4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),   150
     5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,IW,IEST,ND,NFIRE,NHIT  160
     6,NSCARE,ICCORD,IADAPT                                               170
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C      TAKE THE APPROPRIATE GAIN AND STORE IN THE ARRAY GK                    180
C                                                                             190
       DO 1 I=1,N                                                             200
C                                                                             210
       DO 1 J=1,M                                                             220
     1 GK(I,J) = GKS(I,J,K)                                                   230
C                                                                             240
       IF (IADAPT.NE.0) CALL ADAPT                                            250
       IF (IEST.NE.0) GO TO 2                                                 260
       CALL VPROD (HS,XHKKM1,M,N,VTMP)                                        270
       CALL VSUB (Z,VTMP,M,VTMP)                                              280
       CALL VPROD (GK,VTMP,N,M,VTMP)                                          290
       CALL VADD (XHKKM1,VTMP,N,XHKK)                                         300
C                                                                             310
C      XHAT(K/K) HAS BEEN COMPUTED AND STORED IN THE ARRAY XHKK               320
       CALL VPROD (PHIS,XHKK,N,N,XHKKM1)                                      330
C                                                                             340
C      XHAT(K+1/K) HAS BEEN COMPUTED AND STORED IN THE ARRAY XHKKM1           350
       RETURN                                                                 360
     2 CONTINUE                                                               370
C      IF STANDARD EQUATIONS ARE NOT TO BE USED, THE APPROPRIATE              380
C      EQUATIONS MUST BE INSERTED HERE BY THE USER.                           390
       RETURN                                                                 400
       END                                                                    410
C
C
C
       SUBROUTINE GAIN                                                         10
C
       REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI             20
       COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,      30
     19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9       40
     2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),       50
     3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9       60
     4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),       70
     5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,IW,IEST,ND,NFIRE,NHIT       80
     6,NSCARE,ICOORD,IADAPT                                                   90
C                                                                            100
C      G(K) = P(K/K-1)*HT*(H*P(K/K-1)*HT + R)                                110
       CALL TRANS (H,M,N,TEMP2)                                              120
       CALL PROD (PKKM1,TEMP2,N,N,M,TEMP)                                    130
       CALL PROD (H,TEMP,M,N,M,TEMP1)                                        140
       CALL ADD (TEMP1,R,M,M,TEMP1)                                          150
       IF (M.EQ.1) GO TO 2                                                   160
       MD = ND                                                               170
       CALL GAUSS3 (M,EPS,TEMP1,TEMP2,KER,MD)                                180
       CALL PROD (TEMP,TEMP2,N,M,M,G)                                        190
C                                                                            200
```

```
C       NOTE HERE  PKK(I,J) = P(K/K)  WHERE                                  210
C       P(K/K) = (I-G(K)*H)*P(K/K-1)                                         220
      1 CALL PROD (G,H,N,M,N,TEMP)                                           230
        CALL SUB (EI,TEMP,N,N,TEMP2)                                         240
        CALL PROD (TEMP2,PKKM1,N,N,N,PKK)                                    250
C                                                                            260
C       NOTE HERE  PKKM1(I,J) = P(K/K-1)  WHERE                              270
C       P(K/K-1)= PHI*P(K-1/K-1)*PHIT + Q                                    280
        CALL TRANS (PHI,N,N,TEMP2)                                           290
        CALL PROD (PKK,TEMP2,N,N,N,TEMP)                                     300
        CALL PROD (PHI,TEMP,N,N,N,TEMP1)                                     310
        CALL ADD (TEMP1,Q,N,N,PKKM1)                                         320
        RETURN                                                               330
C                                                                            340
      2 DO 3 I=1,N                                                           350
      3 G(I,1) = TEMP(I,1)/TEMP1(1,1)                                        360
C                                                                            370
        GO TO 1                                                              380
        END                                                                  390
C
CC
C
        SUBROUTINE MEAS                                                      10
C
C       THIS SUBROUTINE STARTS WITH THE TRUE STATE VALUE XS                  20
C       AND ADDS ZERO-MEAN WHITE GAUSSIAN NOISE TO H*XS TO                   30
C       GENERATE A NOISY VERCTOR OF MEASUREMENTS Z.                          40
CC                                                                           50
C                                                                            60
        REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI          70
        COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,   80
       19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9   90
       2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),   100
       3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9   110
       4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),   120
       5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,IW,IEST,ND,NFIRE,NHIT   130
       6,NSCARE,ICOORD,IADAPT                                                140
C                                                                            150
CC         THE FOLLOWING ALGORITHM IS VALID FOR 1,2, OR 3 DIMENSIONAL        160
C          SYSTEMS WHEN MEASUREMENTS ARE TAKEN OF POSITION ONLY. FOR OTHER    170
C          CASES IT MUST BE REVISED.                                         180
C                                                                            190
        CALL VPROD (HS,X,M,N,Z)                                              200
        IF (M-2) 1,2,3                                                       210
      1 Z(2) = 0.0                                                           220
      2 Z(3) = 0.0                                                           230
      3 Z1 = SQRT(Z(1)**2+Z(2)**2+Z(3)**2)                                   240
        IF (M-2) 6,5,4                                                       250
```

```
    4 Z(3) = ATAN2(Z(3),Z(1))                                                260
    5 Z(2) = ATAN2(Z(2),Z(1))                                                270
    6 Z(1) = Z1                                                              280
      CALL SNORM (IV,V,M)                                                    290
C                                                                            300
      DO 7 I=1,M                                                             310
    7 V(I) = SIGV(I)*V(I)                                                    320
C                                                                            330
      CALL VADD (Z,V,M,Z)                                                    340
      ALFA = Z(1)*COS(Z(2))*COS(Z(3))                                        350
      BETA = Z(1)*SIN(Z(2))*COS(Z(3))                                        360
      GAMMO = Z(1)*SIN(Z(3))                                                 370
      Z(1) = ALFA                                                            380
      Z(2) = BETA                                                            390
      Z(3) = GAMMO                                                           400
      END                                                                    410
C
CCC
      SUBROUTINE QMAT                                                         10
C                                                                             20
CCC   THIS SUBROUTINE COMPUTES THE MATRIX Q FROM THE EQUATION                 30
CCCCC                                                                         40
CCCCC    Q=GAMMA* E(W*WT) * GAMMAT                                            50
CCCCC                                                                         60
CCCCC DOUBLE PRECISION ARITHMETIC IS USED                                     70
CCC                                                                           80
C                                                                             90
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI            100
      COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,     110
     19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9    120
     2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),    130
     3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9    140
     4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),     150
     5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRC,IXZ,IV,IW,IEST,ND,NFIRE,NHIT    160
     6,NSCARE,ICOORD,IADAPT                                                 170
C                                                                           180
CC                                                                          190
      CALL PROD (GAMMA,COVW,N,IN,IN,TEMP)                                   200
      CALL TRANS (GAMMA,N,IN,TEMP1)                                         210
      CALL PROD (TEMP,TEMP1,N,IN,N,Q)                                       220
      RETURN                                                                230
      END                                                                   240
C
CCC
C
      SUBROUTINE QON                                                          10
C                                                                             20
```

```
C      IF Q IS TO BE COMPUTED ON-LINE (IFLQ.NE.0) IT IS DONE                    30
CC     IN THIS SUBROUTINE                                                       40
C                                                                               50
       REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI              60
       COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,       70
     19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9        80
     2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),        90
     3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9       100
     4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),        110
     5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,IW,IEST,ND,NFIRE,NHIT       120
     6,NSCARE,ICOORD,IADAPT                                                     130
C                                                                               140
CC     THE APPROPRIATE STATEMENTS FOR COMPUTING Q ON-LINE MUST                  150
CC     BE INSERTED HERE BY THE USER                                             160
C                                                                               170
       RETURN                                                                   180
       END                                                                      190
C
CC
C      SUBROUTINE RON                                                           10
C                                                                               20
CCC    IF R IS TO BE COMPUTED ON-LINE (IFLR.NE.0) IT IS DONE                    30
CC     IN THIS SUBROUTINE                                                       40
C                                                                               50
       REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI              60
       COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,       70
     19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9        80
     2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),        90
     3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9       100
     4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),        110
     5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,IW,IEST,ND,NFIRE,NHIT       120
     6,NSCARE,ICOORD,IADAPT                                                     130
C                                                                               140
CC     THE APPROPRIATE STATEMENTS FOR COMPUTING R ON-LINE MUST                  150
CC     BE INSERTED HERE BY THE USER                                             160
C                                                                               170
       RETURN                                                                   180
       END                                                                      190
C
CC
C      SUBROUTINE STAT                                                          10
C
CC     THIS SUBROUTINE  COMPUTES RUNNING SUMS USED IN DETERMINING THE           20
CC     SAMPLE STATISTICS OF TRACK AND ESTIMATION ERRORS. IN THE DEFAULT         30
C      OPTION (ISTAT.EQ.0) THE STATISTICS TO BE COMPUTED ARE MEAN OF            40
C      TRACK, MEAN OF ESTIMATION ERROR AND VARIANCE OF ESTIMATION               50
```

```
C      ERROR. IF(ISTAT.NE.0) THE OFF-DIAGONAL TERMS IN THE COVARIANCE OF        60
C      ESTIMATION ERROR MATRIX ARE ALSO COMPUTED.                               70
C
       REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI              80
       COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,       90
      19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9      100
      2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),      110
      3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9      120
      4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),       130
      5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,IW,IEST,ND,NFIRE,NHIT      140
      6,NSCARE,ICOORD,IADAPT                                                   150
       DIMENSION EXH(9)                                                        160
       IF (ITRK.NE.1) GO TO 2                                                  170
       IF (ITER.NE.1) GO TO 4                                                  180
C                                                                              190
       DO 1 J=1,N                                                              200
     1 XM(J,K) = XS(J,K)                                                       210
C                                                                              220
       GO TO 4                                                                 230
     2 CONTINUE                                                                240
C                                                                              250
       DO 3 J=1,N                                                              260
     3 XM(J,K) = XM(J,K)+XS(J,K)                                               270
C                                                                              280
     4 CONTINUE                                                                290
C                                                                              300
       DO 5 J=1,N                                                              310
       EXH(J) = XHKK(J)-XS(J,K)                                                320
       ERR(J,K) = ERR(J,K)+EXH(J)                                              330
     5 VAR(J,J,K) = VAR(J,J,K)+EXH(J)**2                                       340
C                                                                              350
       IF (ISTAT.EQ.0) RETURN                                                  360
C                                                                              370
       DO 6 L=2,N                                                              380
       LM1 = L-1                                                               390
C                                                                              400
       DO 6 J=1,LM1                                                            410
     6 VAR(L,J,K) = VAR(L,J,K)+EXH(L)*EXH(J)                                   420
C                                                                              430
       RETURN                                                                  440
       END                                                                     450
C
CC
C
       SUBROUTINE TRACK                                                        10
C
CC     IF TRACK IS TO BE GENERATED ON-LINE IT IS DONE IN THIS SUBROUTINE       20
C      IN THE DEFAULT OPTION (ITRK.EQ.0) THE TRACK IS GENERATED                30
```

```
C      FROM THE STANDARD LINEAR DIFFERENCE EQUATION                          40
CC                                                                           50
CC     X(K+1)=PHI*X(K)+GAMMA*W(K)                                            60
C                                                                            70
       REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI           80
       COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,    90
      19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9   100
      2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),   110
      3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9   120
      4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),   130
      5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,IW,IEST,ND,NFIRE,NHIT   140
      6,NSCARE,ICOORD,IADAPT                                                150
       DIMENSION W(3)                                                       160
C      ITRK NE.0 OR 1 -- SEVERAL TRACKS GENERATED,BUT NOT FROM STD.          170
CC                       LINEAR EQS.                                         180
CC         = 0 -- SEVERAL TRACKS GENERATED FROM STD LINEAR EQS               190
C          = 1 -- ONLY ONE TRACK IS USED                                     200
       IF (ITRK.NE.0) GO TO 4                                                210
       CALL SNORM (IW,W,IN)                                                  220
C      CONVERT EACH N(0,1) R.V. TO N(0,SIGW(I)) R.V.                         230
CC                                                                           240
       DO 1 I=1,IN                                                           250
     1 W(I) = SIGW(I)*W(I)                                                   260
CC                                                                           270
CC                                                                           280
       DO 2 I=1,N                                                            290
     2 X(I) = XS(I,K)                                                        300
C                                                                            310
       CALL VPROD (GAMMAS,W,N,IN,W)                                          320
       CALL VPROD (PHIS,X,N,N,VTMP)                                          330
       CALL VADD (VTMP,W,N,VTMP)                                             340
C                                                                            350
       DO 3 I=1,N                                                            360
     3 XS(I,K+1) = VTMP(I)                                                   370
C                                                                            380
CC     NEW VALUE OF X HAS BEEN COMPUTED AND STORED IN THE ARRAY XS           390
       RETURN                                                                400
     4 IF (ITRK.NE.1) GO TO 6                                                410
C        IF(ITRK.EQ.1) THE USER MUST INSERT HERE THE STATEMENTS REQUIRED      420
CC     TO GENERATE A SINGLE TRAJECTORY AND STORE IT IN THE ARRAY             430
CC     XS(I,K),I=1,N,K=2,NSAM (NOTE THAT IF A SINGLE TRAJECTORY IS TO BE     440
CC     GENERATED, THE INITIAL CONDITION HAS BEEN READ IN AND STORED          450
CC     IN XS(I,1),I=1,N)                                                     460
CC                                                                           470
       DO 5 K=2,NSAM                                                         480
       EKM1 = K-1                                                            490
       T = 0.5*EKM1                                                          500
       XS(1,K) = 13.-0.0173205*T                                             510
```

```
      XS(2,K) = -0.0173205                                                  520
      XS(3,K) = 7.7-0.01*T                                                  530
    5 XS(4,K) = -0.01                                                       540
C                                                                           550
      RETURN                                                                560
    6 CONTINUE                                                              570
C     IF THIS POINT IS REACHED, ITRK NOT EQUAL 0 OR 1 INDICATING THAT       580
C     SEVERAL TRACKS ARE TO BE GENERATED, BUT NOT BY USING THE STD.         590
C     LINEAR DIFFERENCE EQS..THE USER MUST SUPPLY THE APPROPRIATE           600
C     STATEMENTS HERE.                                                      610
      RETURN                                                                620
      END                                                                   630
C
CC
C
      SUBROUTINE XZERO                                                       10
C
CC    THIS SUBROUTINE GENERATES THE INITIAL STATE VALUE FROM A NORMAL        20
CC    RANDOM NUMBER GENERATOR. IT IS ASSUMED THAT THE INITIAL STATE          30
CC    HAS COMPONENTS THAT ARE INDEPENDENT                                    40
C
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI            50
      COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,     60
     19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9    70
     2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),    80
     3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9    90
     4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),    100
     5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRC,IXZ,IV,IW,IEST,ND,NFIRE,NHIT    110
     6,NSCARE,ICOORD,IADAPT                                                 120
      CALL SNORM (IXZ,X,N)                                                  130
C                                                                           140
      DO 1 I=1,N                                                            150
    1 XS(I,1) = SIGXZ(I)*X(I)+XZMEAN(I)                                     160
C                                                                           170
      RETURN                                                                180
      END                                                                   190
C
CC
C
      SUBROUTINE ADD (A,B,N,M,C)                                             10
C
CC    THIS SUBROUTINE ADDS THE NXM MATRICES A AND B, STORING THE             20
C     RESULT IN C                                                            30
      REAL *8A,B,C                                                           40
      DIMENSION A(9,9), B(9,9), C(9,9)                                       50
C                                                                            60
      DO 1 I=1,N                                                             70
C                                                                            80
```

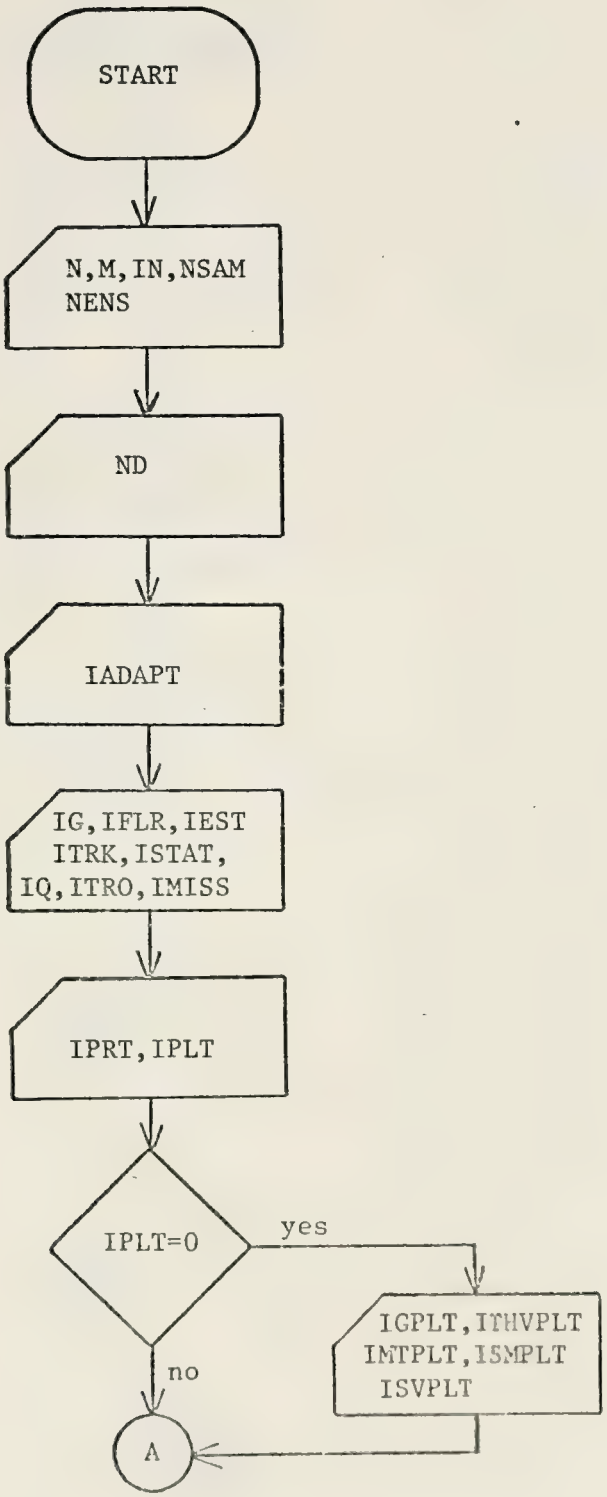
```
      DO 1 J=1,M                                                        90
    1 C(I,J) = A(I,J)+B(I,J)                                           100
C                                                                      110
      RETURN                                                           120
      END                                                              130
C
CC
C
      SUBROUTINE MREAD (A,N,M)                                         10
C
C     THIS SUBROUTINE READS AN NXM MATRIX A ACCORDING TO THE FORMAT    20
C     8D10.5. THE ENTRIES IN THE FIRST ROW OF A ARE READ FIRST, THEN   30
C     THE ENTRIES IN THE SECOND ROW, AND SO ON.                        40
C
      REAL *8A                                                         50
      DIMENSION A(9,9)                                                 60
C                                                                      70
      DO 1 I=1,N                                                       80
    1 READ (5,2) (A(I,J),J=1,M)                                        90
C                                                                     100
    2 FORMAT (8F10.0)                                                 110
      RETURN                                                          120
      END                                                             130
C
CC
C
      SUBROUTINE MWRITE (A,N,M)                                        10
C
C     THIS SUBROUTINE WRITES THE ENTRIES OF THE NXM MATRIX A           20
      REAL *8A                                                         30
      DIMENSION A(9,9)                                                 40
C                                                                      50
      DO 1 I=1,N                                                       60
    1 WRITE (6,2) (A(I,J),J=1,M)                                       70
C                                                                      80
    2 FORMAT (9(2X,1PE12.5))                                           90
      RETURN                                                          100
      END                                                             110
C
CC
C
      SUBROUTINE PROD (A,B,N,M,L,C)                                    10
C
C     THIS SUBROUTINE COMPUTES THE MATRIX PRODUCT AB AND STORES THE    20
C     RESULT IN C                                                      30
C     A = NXM,    B = MXL,    C = NXL                                  40
      REAL *8A,B,C,T                                                   50
      DIMENSION A(9,9), B(9,9), C(9,9), T(9,9)                         60
```

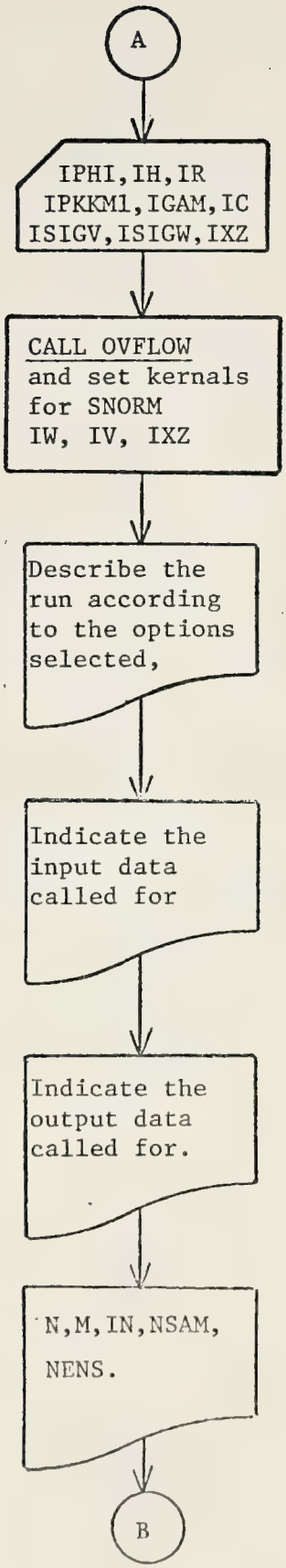
```
C                                                                       70
      DO 1 I=1,N                                                        80
C                                                                       90
      DO 1 J=1,L                                                       100
    1 T(I,J) = 0.0                                                     110
C                                                                      120
C                                                                      130
      DO 2 I=1,N                                                       140
C                                                                      150
      DO 2 J=1,L                                                       160
C                                                                      170
      DO 2 K=1,M                                                       180
    2 T(I,J) = T(I,J)+A(I,K)*B(K,J)                                    190
C                                                                      200
C                                                                      210
      DO 3 I=1,N                                                       220
C                                                                      230
      DO 3 J=1,L                                                       240
    3 C(I,J) = T(I,J)                                                  250
C                                                                      260
      RETURN                                                           270
      END                                                              280
C
CC
C
      SUBROUTINE SUB (A,B,N,M,C)                                        10
C
C     THIS SUBROUTINE SUBTRACTS THE NXM MATRIX B FROM THE NXM MATRIX    20
C     A AND STORES THE RESULT IN C                                      30
      REAL *8A,B,C                                                      40
      DIMENSION A(9,9), B(9,9), C(9,9)                                  50
C                                                                       60
      DO 1 I=1,N                                                        70
C                                                                       80
      DO 1 J=1,M                                                        90
    1 C(I,J) = A(I,J)-B(I,J)                                           100
C                                                                      110
      RETURN                                                           120
      END                                                              130
C
CC
C
      SUBROUTINE TRANS (A,N,M,C)                                        10
C
C     THIS SUBROUTINE FORMS THE MATRIX TRANSPOSE OF A STORING THE       20
C     RESULT IN C                                                       30
C     A = NXM,    C = MXN                                               40
C
```

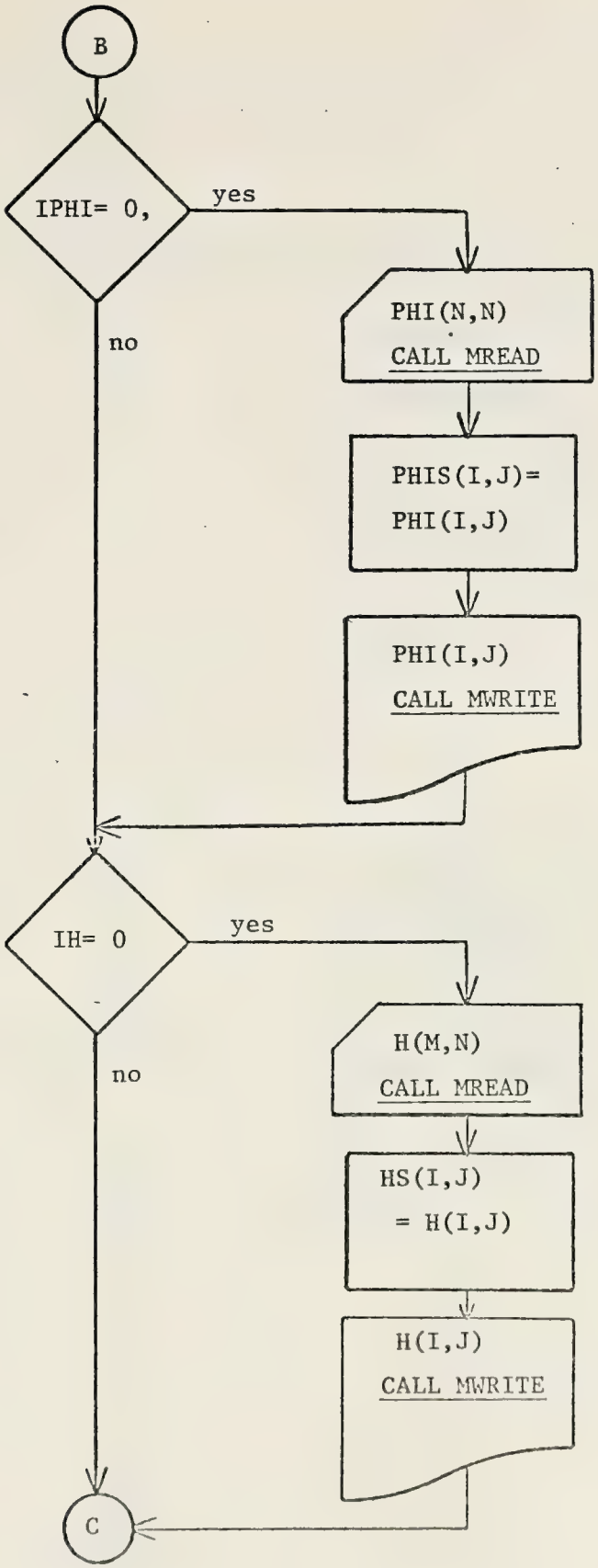
```
      REAL *8A,C                                                        50
      DIMENSION A(9,9), C(9,9)                                          60
C                                                                       70
      DO 1 I=1,N                                                        80
C                                                                       90
      DO 1 J=1,M                                                       100
    1 C(J,I) = A(I,J)                                                  110
C                                                                      120
      RETURN                                                           130
      END                                                              140
C
CC
C
      SUBROUTINE VADD (X,Y,N,Z)                                         10
C
CC    THIS SUBROUTINE COMPUTES THE SUM OF THE N-VECTORS X AND           20
CC    Y AND STORES THE RESULT IN THE N-VECTOR Z                         30
C                                                                       40
      REAL *4X(9),Y(9),Z(9)                                             50
C                                                                       60
      DO 1 I=1,N                                                        70
    1 Z(I) = X(I)+Y(I)                                                  80
C                                                                       90
      RETURN                                                           100
      END                                                              110
C
CC
C
      SUBROUTINE VPROD (A,X,M,N,Y)                                      10
C
CC    THIS SUBROUTINE COMPUTES THE PRODUCT OF THE MXN MATRIX            20
CC    A AND THE N-VECTOR X AND STORES THE RESULT IN THE                 30
CC    M-VECTOR Y                                                        40
CC                                                                      50
C                                                                       60
      REAL *4A(9,9),X(9),Y(9),T(9)                                      70
C                                                                       80
      DO 1 I=1,M                                                        90
      T(I) = 0.D0                                                      100
C                                                                      110
      DO 1 J=1,N                                                       120
    1 T(I) = T(I)+A(I,J)*X(J)                                          130
C                                                                      140
CC                                                                     150
      DO 2 I=1,M                                                       160
    2 Y(I) = T(I)                                                      170
C                                                                      180
      RETURN                                                           190
```

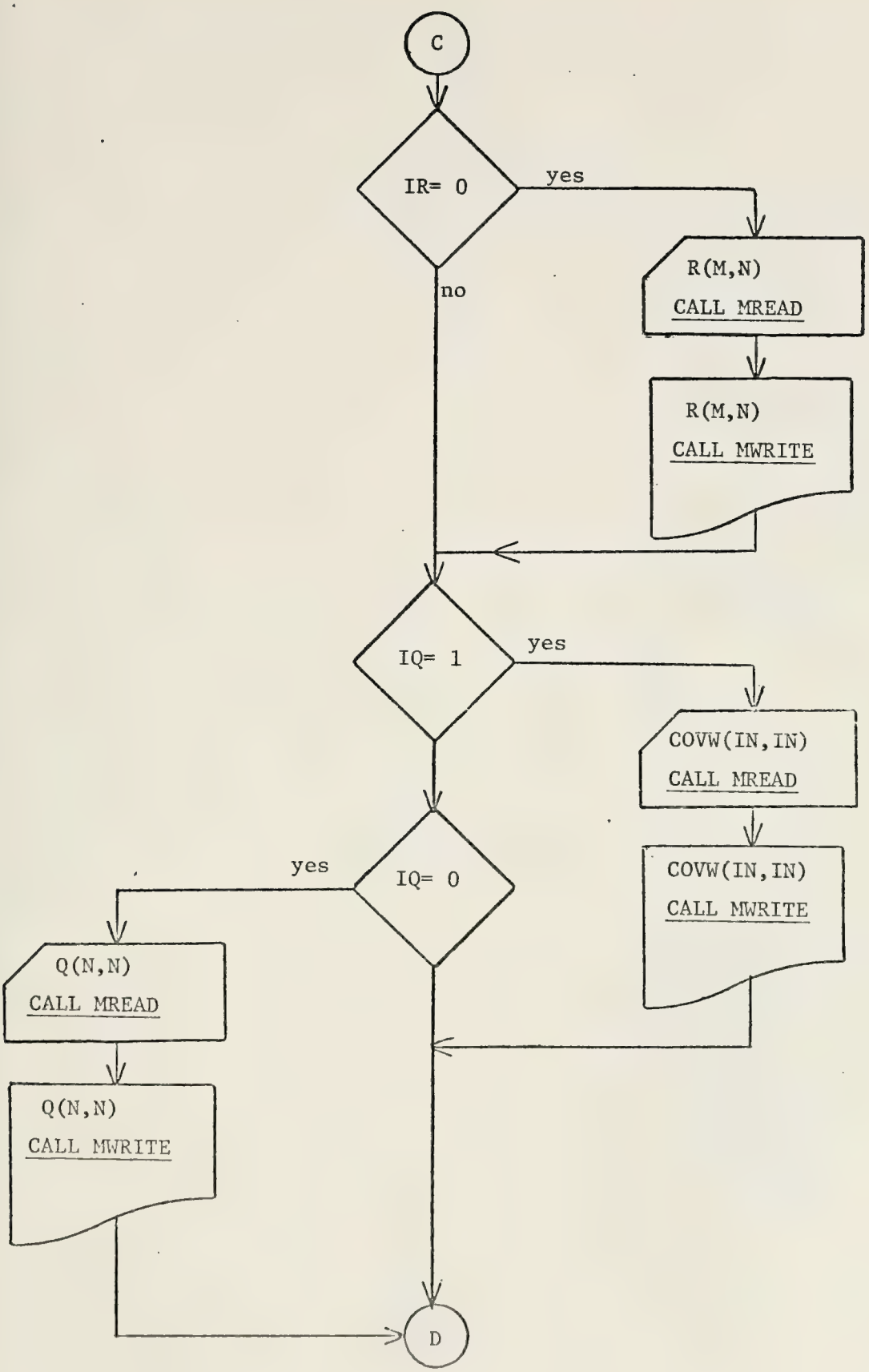
```
      END                                                                200
C
CC
C
      SUBROUTINE VREAD (V,N)                                              10
C                                                                         20
CC    THIS SUBROUTINE READS THE N-DIMENSIONAL S.P. VECTOR V               30
C                                                                         40
      DIMENSION V(9)                                                      50
      READ (5,1) (V(I),I=1,N)                                             60
    1 FORMAT (8F10.0)                                                     70
      RETURN                                                              80
      END                                                                 90
C
CC
C
      SUBROUTINE VSUB (X,Y,N,Z)                                           10
C                                                                         20
CC    THIS SUBROUTINE COMPUTES THE DIFFERENCE X-Y OF THE TWO              30
C     N-VECTORS X & Y AND STORES THE RESULT IN THE N-VECTOR Z             40
C                                                                         50
      REAL *4X(9),Y(9),Z(9)                                               60
C                                                                         70
      DO 1 I=1,N                                                          80
    1 Z(I) = X(I)-Y(I)                                                    90
C                                                                        100
      RETURN                                                             110
      END
C
CC
C
      SUBROUTINE VWRITE (V,N)                                             10
C                                                                         20
CC    THIS SUBROUTINE WRITES THE N-DIMENSIONAL S.P. VECTOR V              30
C                                                                         40
      DIMENSION V(9)                                                      50
      WRITE (6,1) (V(I),I=1,N)                                            60
    1 FORMAT (9(2X,1PE12.5))                                              70
      RETURN                                                              80
      END                                                                 90
```

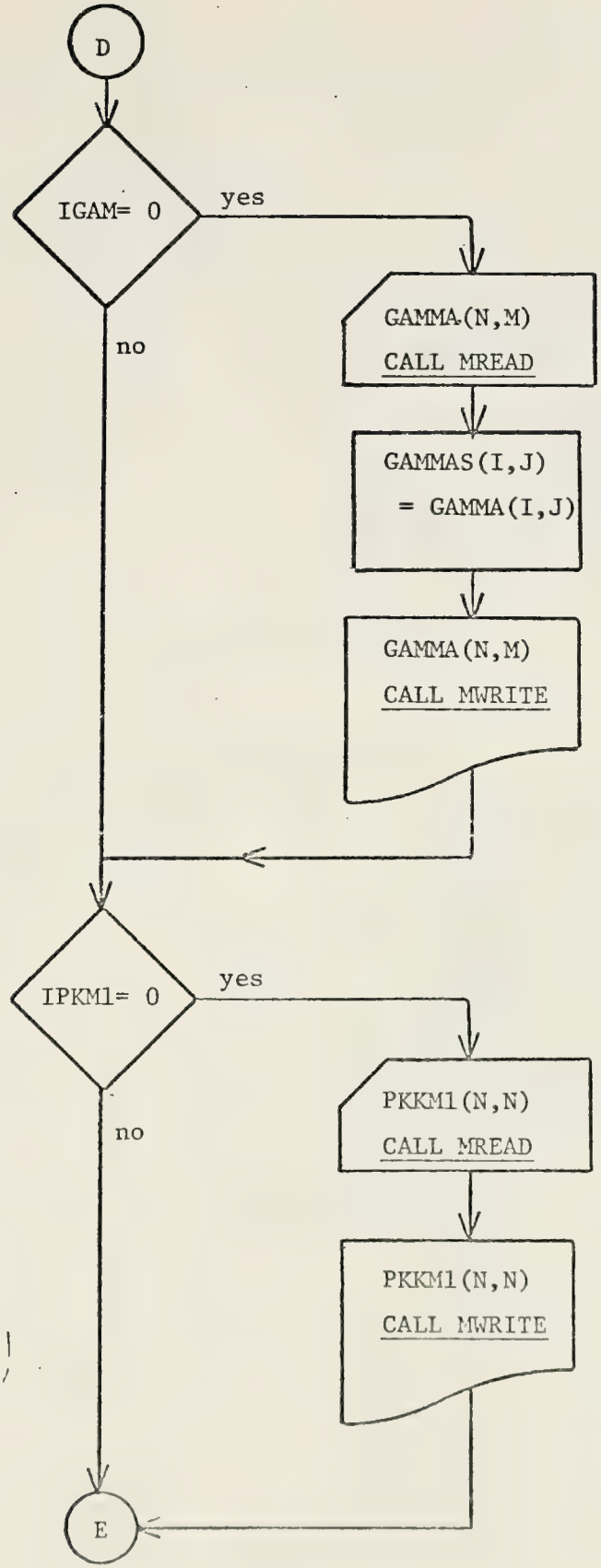
A.3 Flowchart of the Monte-Carlo Simulation Program

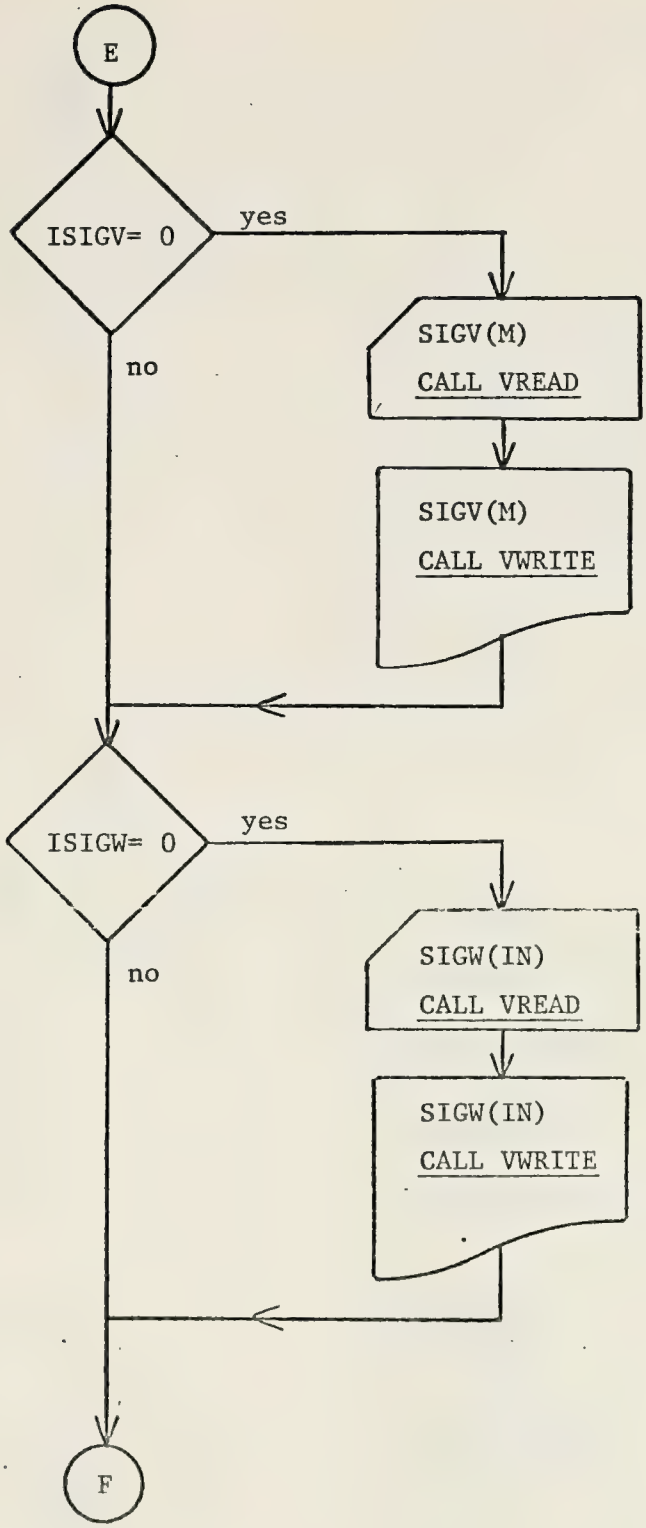


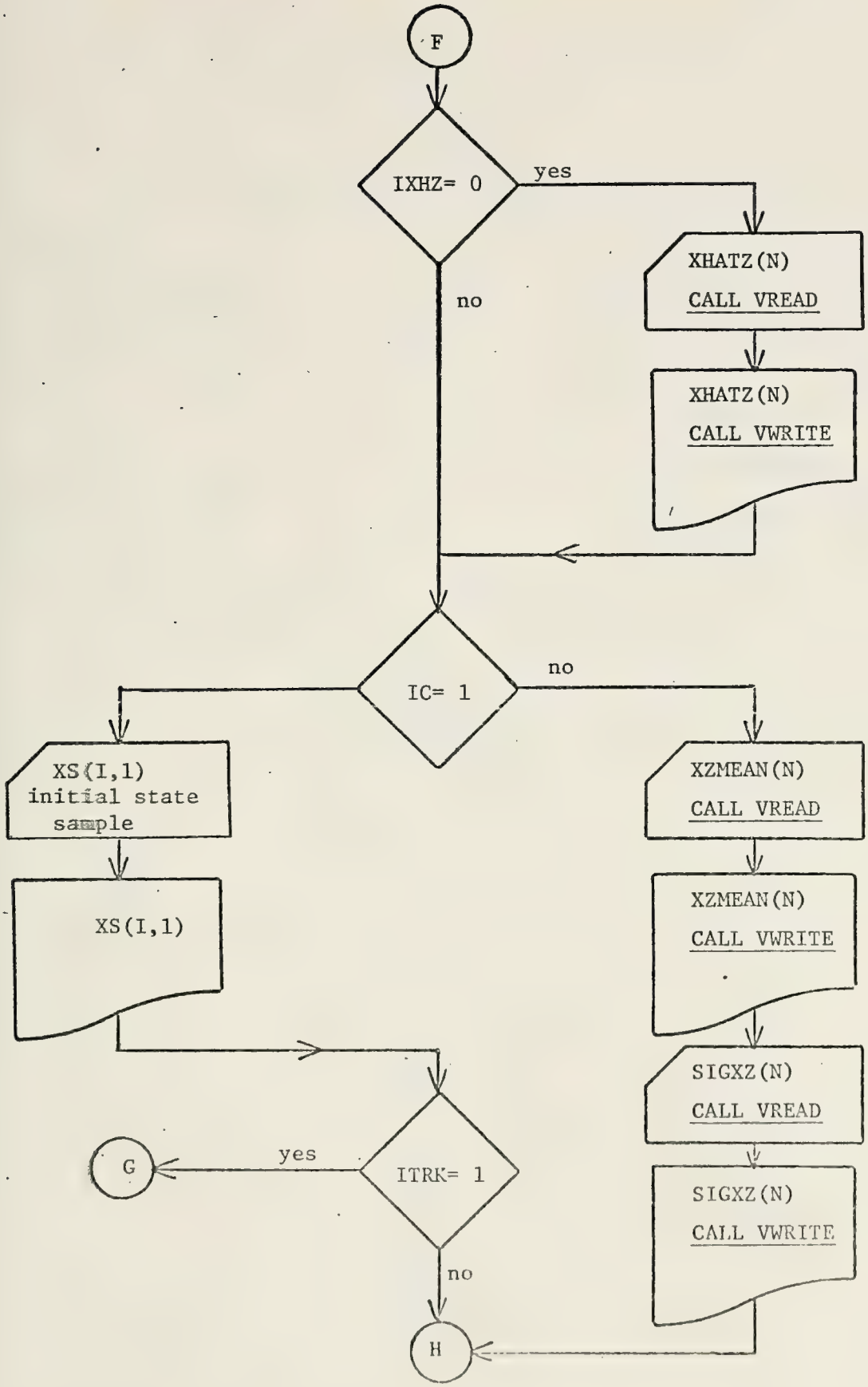

A
IPHI,IH,IR
IPKKM1,IGAM,IC
ISIGV,ISIGW,IXZ
CALL OVFLOW
and set kernals
for SNORM
IW, IV, IXZ
Describe the
run according
to the options
selected,
Indicate the
input data
called for
Indicate the
output data
called for.
N,M,IN,NSAM,
NENS.
B

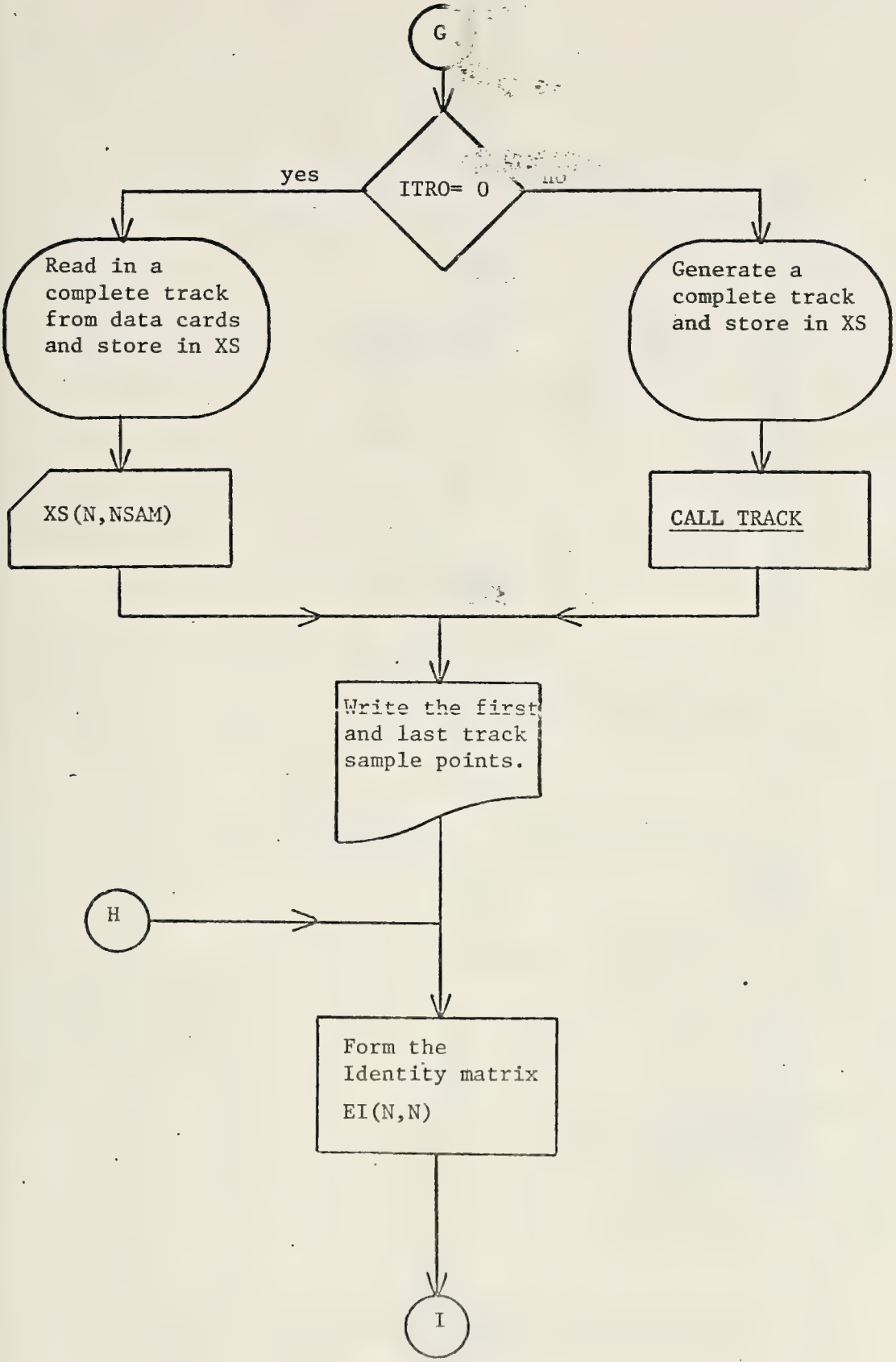

B
IPHI= 0,
yes
no
PHI(N,N)
CALL MREAD
PHIS(I,J)=
PHI(I,J)
PHI(I,J)
CALL MWRITE
IH= 0
yes
no
H(M,N)
CALL MREAD
HS(I,J)
= H(I,J)
H(I,J)
CALL MWRITE
C

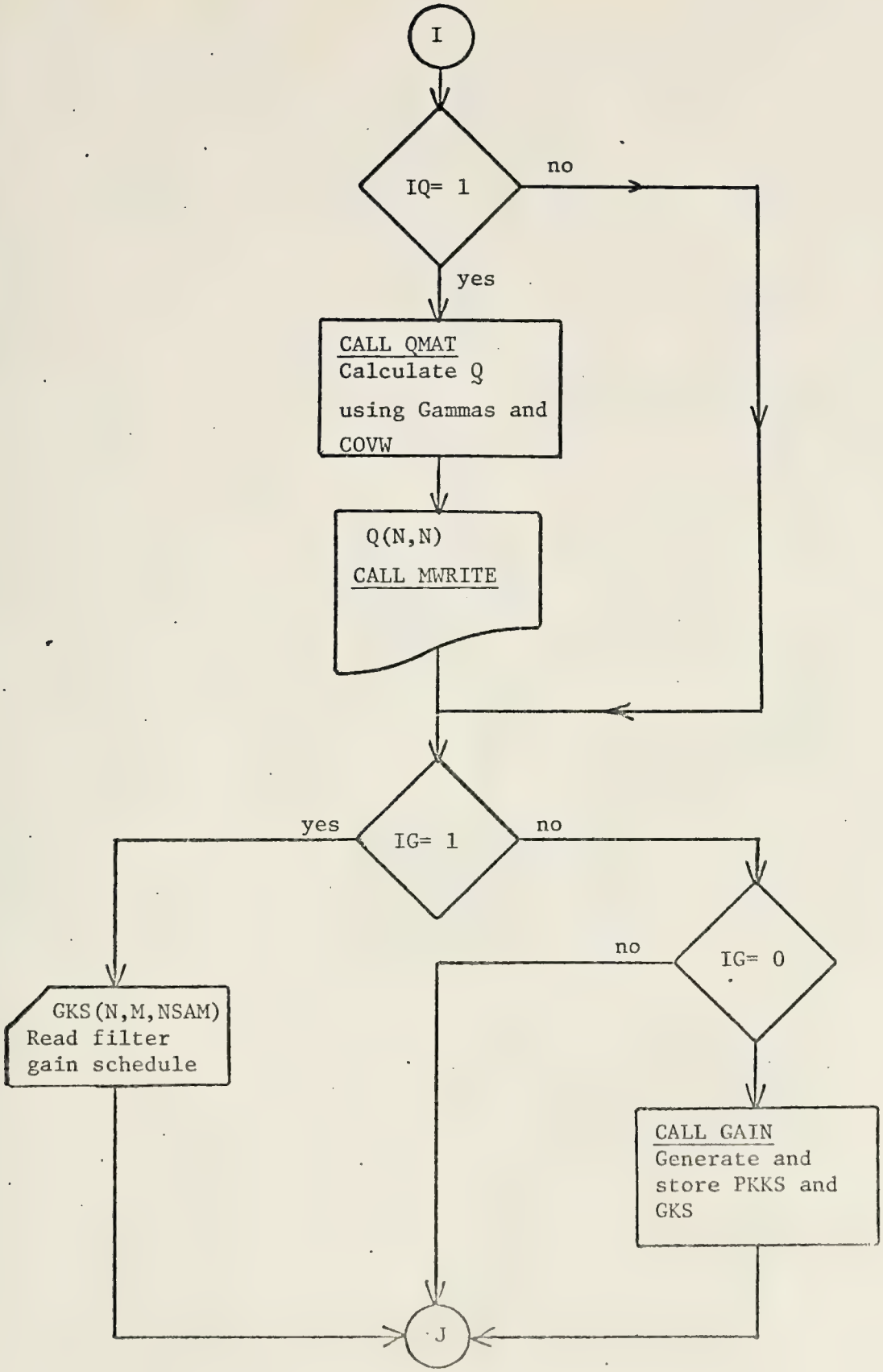

C
IR= 0
yes
no
R(M,N)
CALL MREAD
R(M,N)
CALL MWRITE
IQ= 1
yes
COVW(IN,IN)
CALL MREAD
COVW(IN,IN)
CALL MWRITE
IQ= 0
yes
Q(N,N)
CALL MREAD
Q(N,N)
CALL MWRITE
D

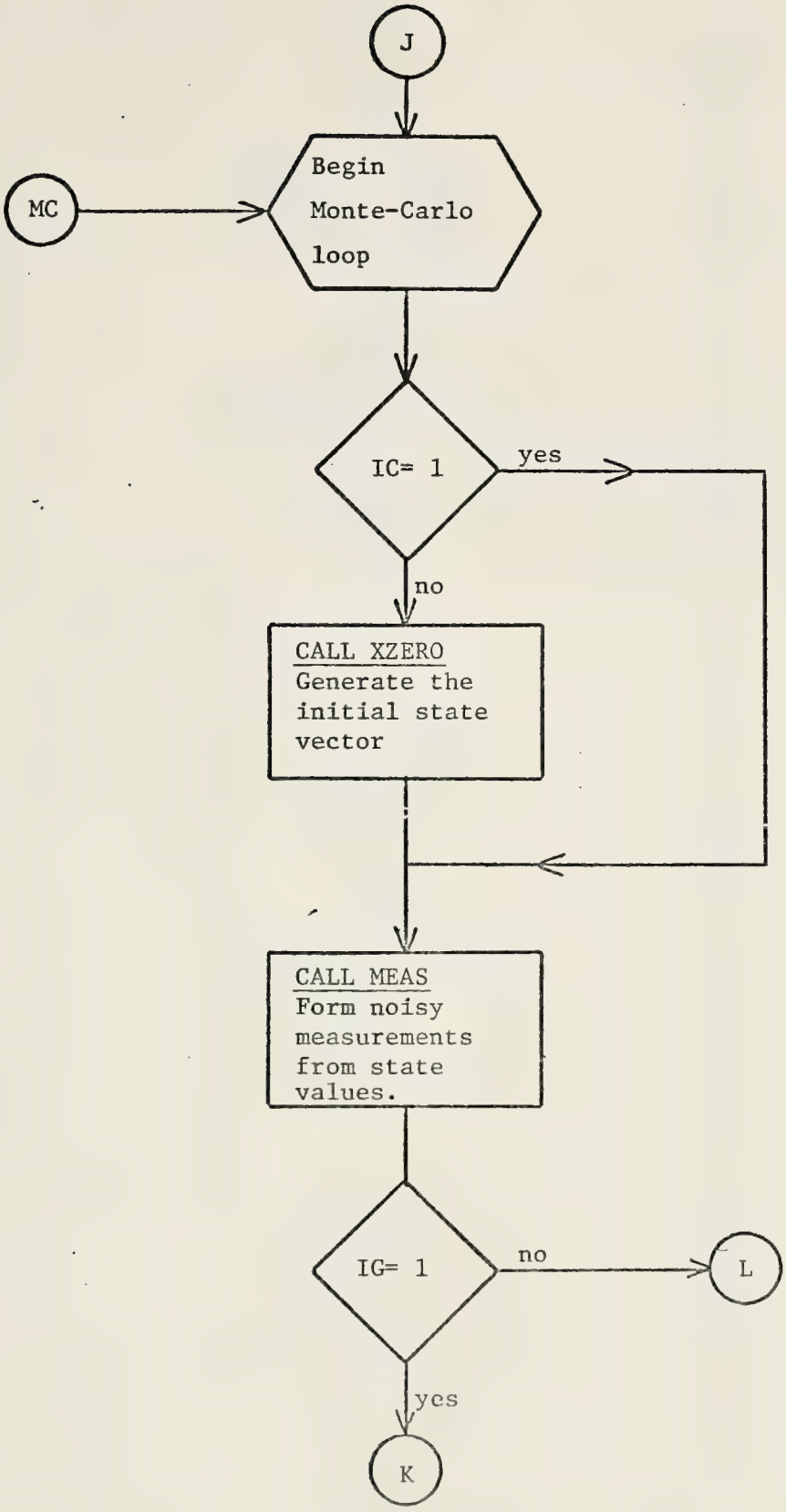

D
IGAM= 0
yes
no
GAMMA(N,M)
CALL MREAD
GAMMAS(I,J)
= GAMMA(I,J)
GAMMA(N,M)
CALL MWRITE
IPKM1= 0
yes
no
PKKM1(N,N)
CALL MREAD
PKKM1(N,N)
CALL MWRITE
E

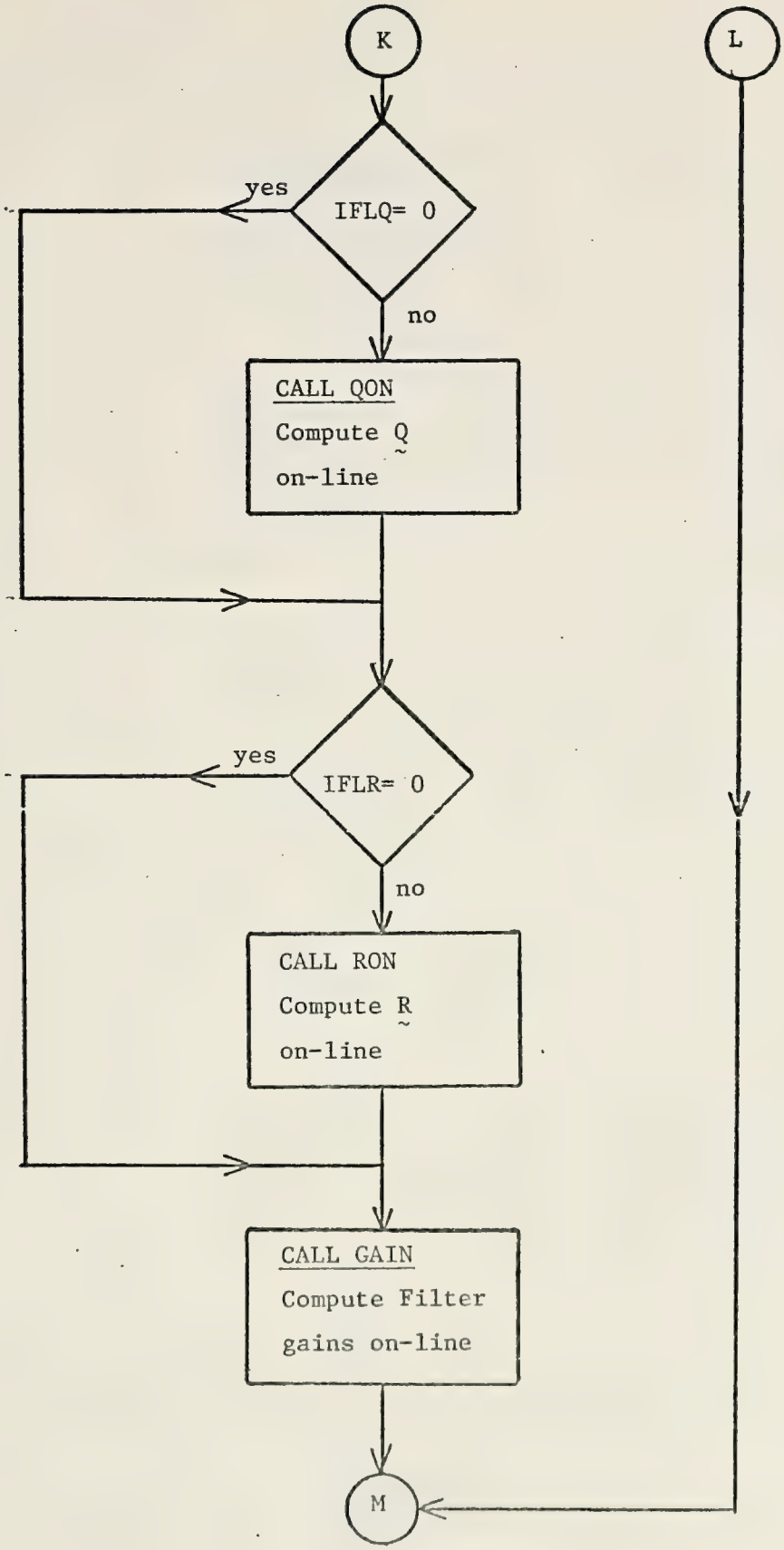

E
ISIGV= 0
yes
no
SIGV(M)
CALL VREAD
SIGV(M)
CALL VWRITE
ISIGW= 0
yes
no
SIGW(IN)
CALL VREAD
SIGW(IN)
CALL VWRITE
F

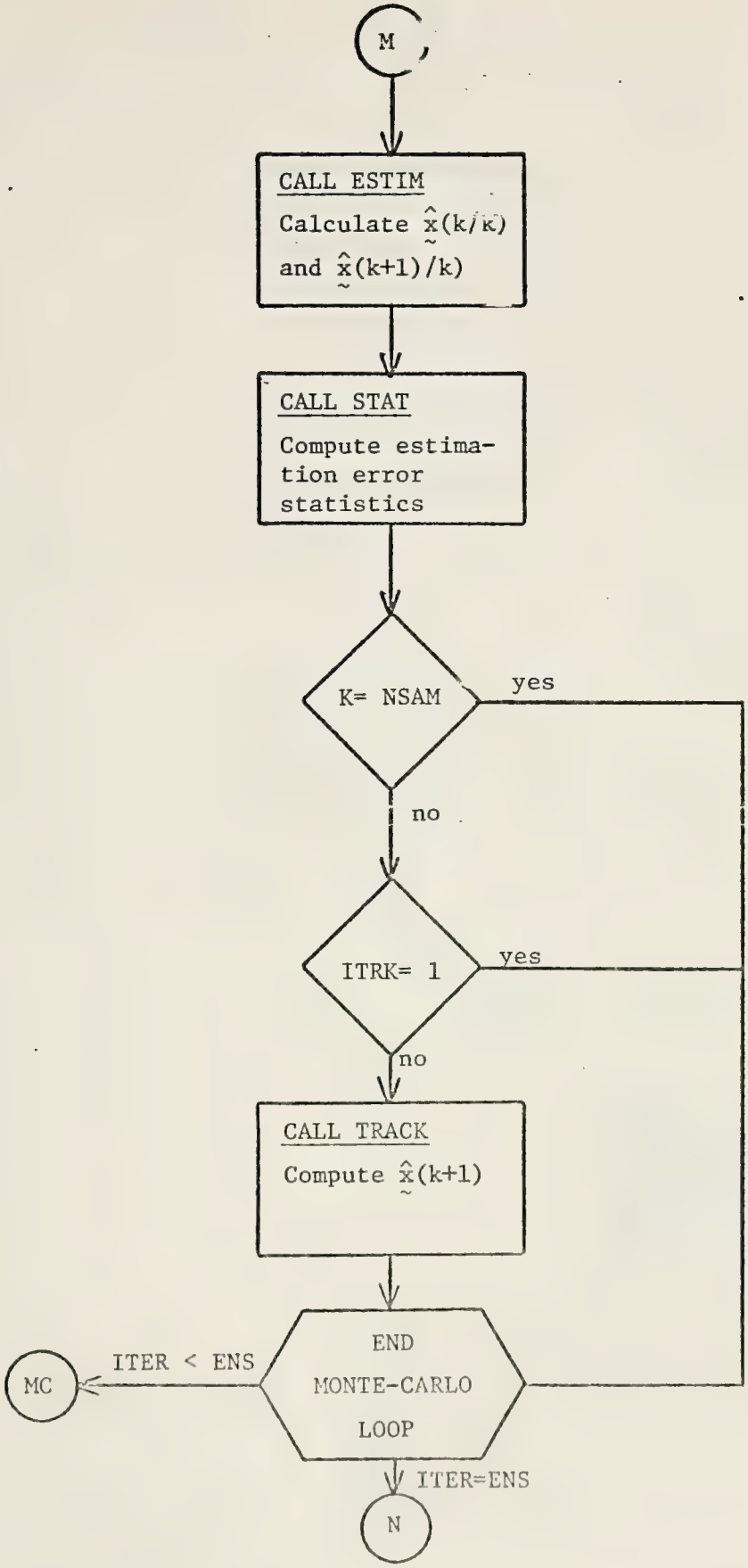

F
IXHZ= 0
yes
no
XHATZ(N)
CALL VREAD
XHATZ(N)
CALL VWRITE
IC= 1
no
XS(I,1)
initial state
sample
XS(I,1)
XZMEAN(N)
CALL VREAD
XZMEAN(N)
CALL VWRITE
SIGXZ(N)
CALL VREAD
SIGXZ(N)
CALL VWRITE
G
yes
ITRK= 1
no
H

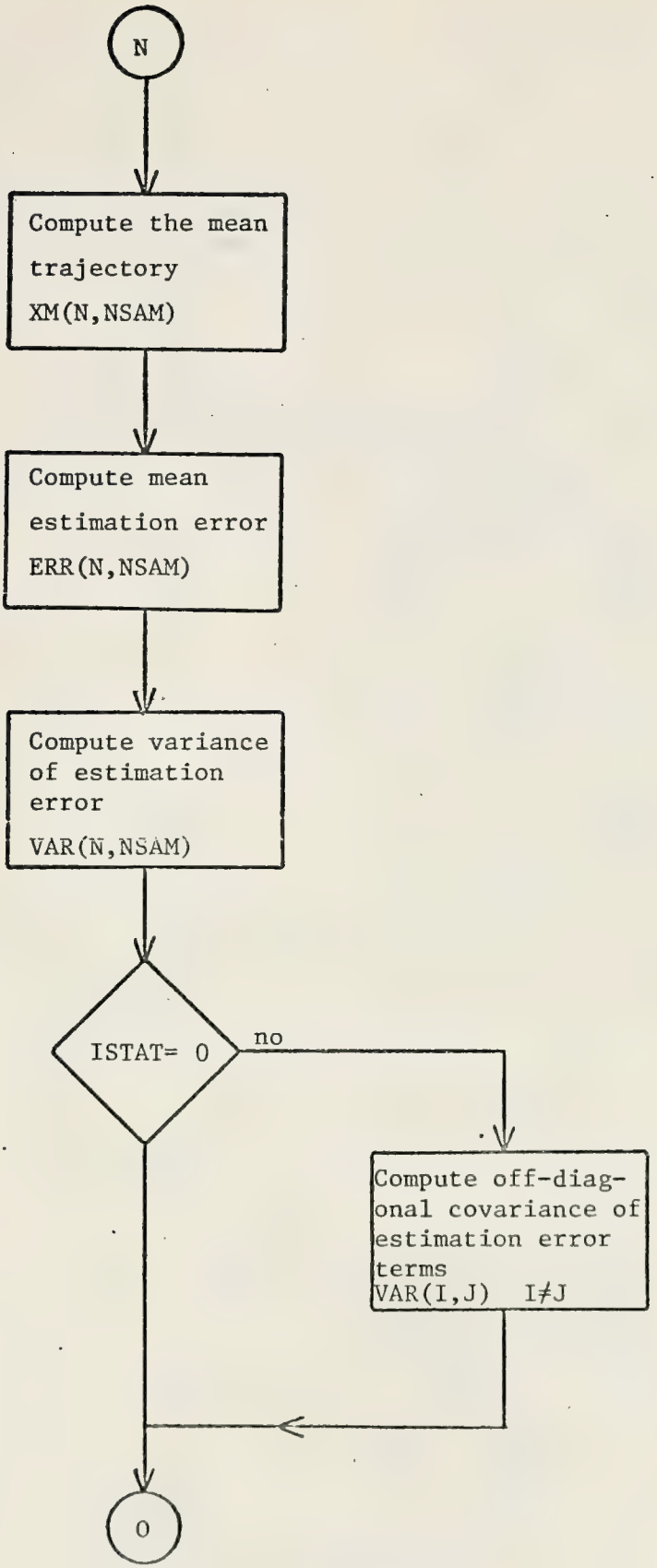

G
ITRO= 0
yes
no
Read in a complete track from data cards and store in XS
Generate a complete track and store in XS
XS(N,NSAM)
CALL TRACK
Write the first and last track sample points.
H
Form the Identity matrix
EI(N,N)
I

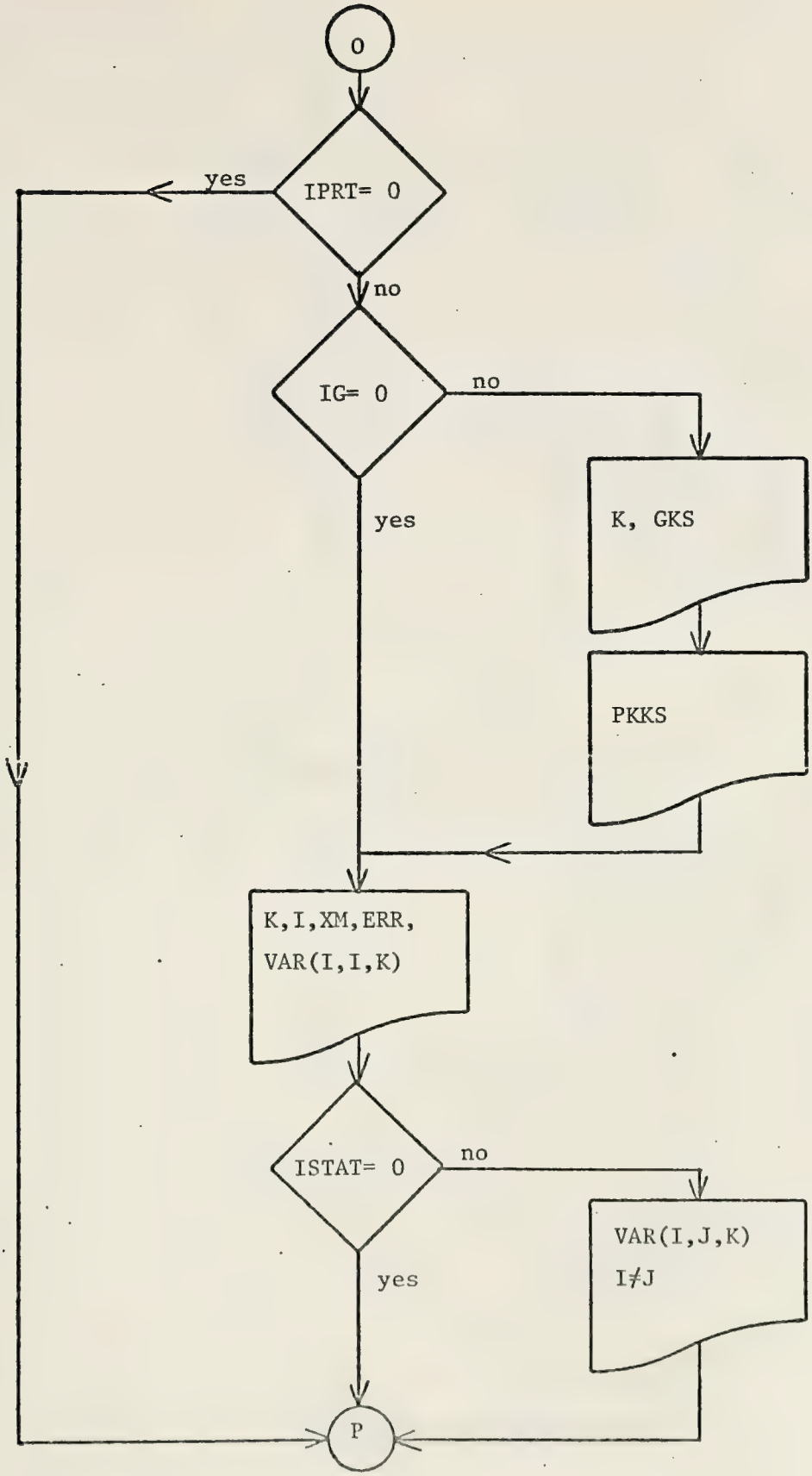

I
IQ= 1
no
yes
CALL QMAT
Calculate Q
using Gammas and
COVW
Q(N,N)
CALL MWRITE
IG= 1
yes
no
IG= 0
no
GKS(N,M,NSAM)
Read filter
gain schedule
CALL GAIN
Generate and
store PKKS and
GKS
J

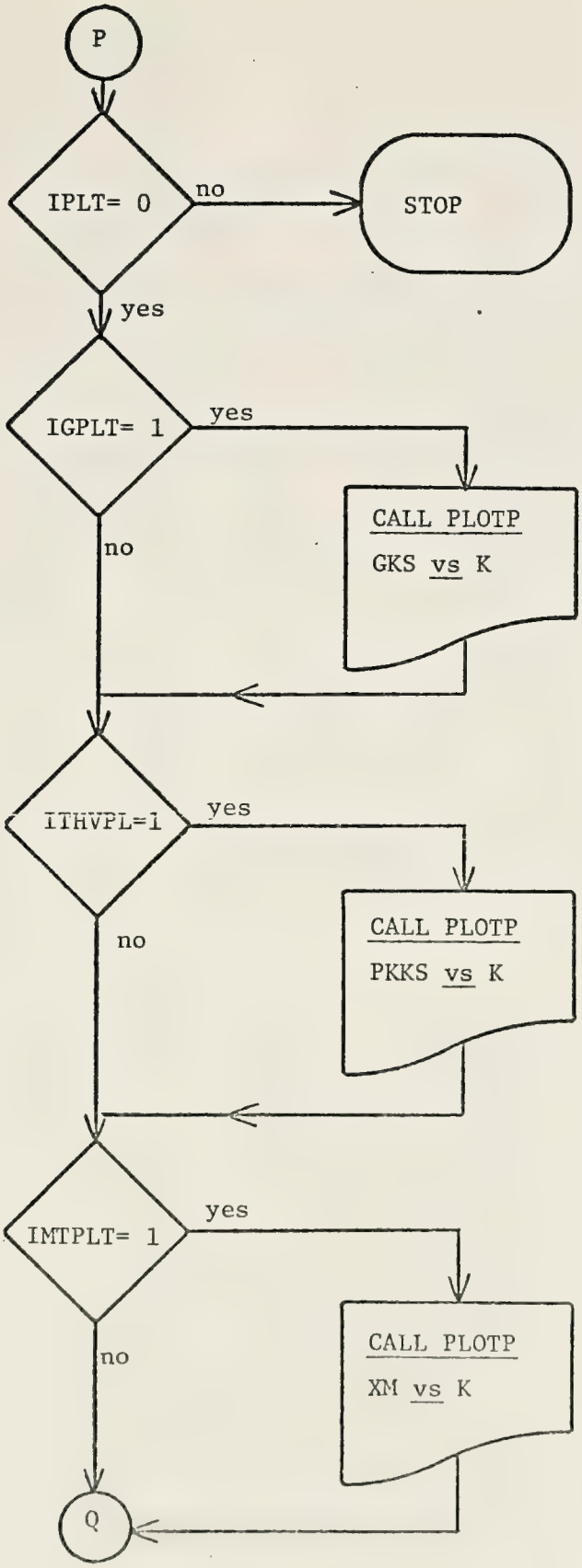

J
MC
Begin
Monte-Carlo
loop
IC= 1
yes
no
CALL XZERO
Generate the
initial state
vector
CALL MEAS
Form noisy
measurements
from state
values.
IG= 1
no
L
yes
K

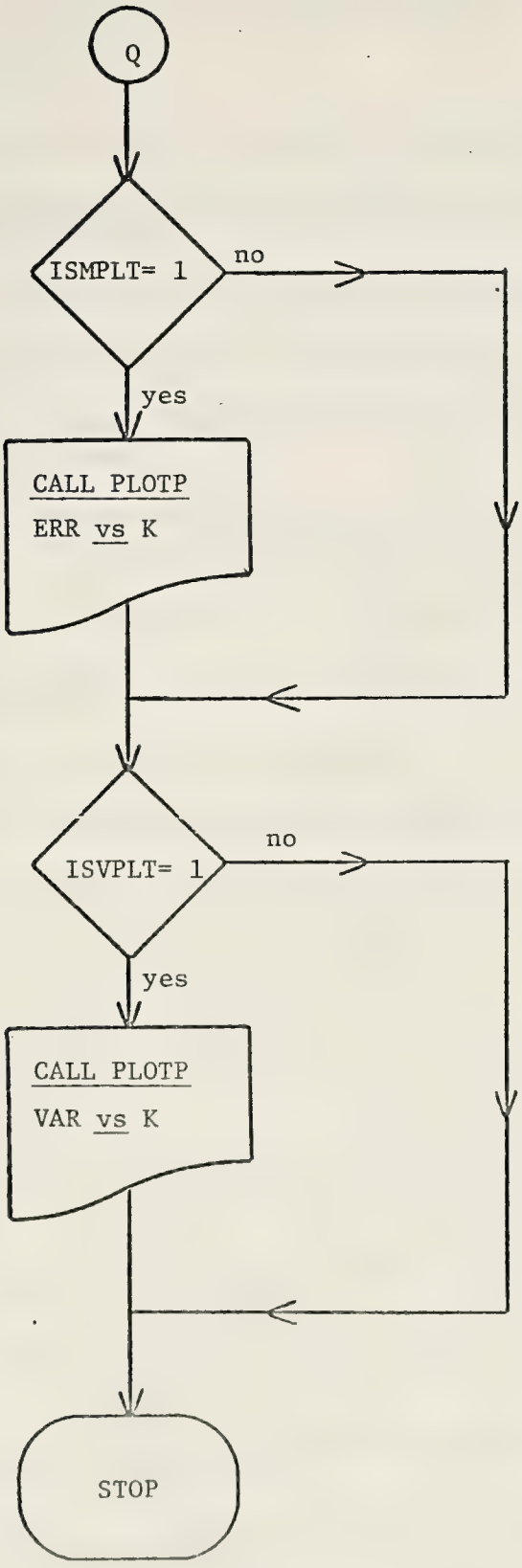

K
L
IFLQ= 0
yes
no
CALL QON
Compute Q
on-line
IFLR= 0
yes
no
CALL RON
Compute R
on-line
CALL GAIN
Compute Filter
gains on-line
M


M
CALL ESTIM
Calculate $\hat{x}(k/k)$
and $\hat{x}(k+1)/k)$
CALL STAT
Compute estimation error statistics
K= NSAM
yes
no
ITRK= 1
yes
no
CALL TRACK
Compute $\hat{x}(k+1)$
END
MONTE-CARLO
LOOP
ITER < ENS
MC
ITER=ENS
N


N
Compute the mean trajectory XM(N,NSAM)
Compute mean estimation error ERR(N,NSAM)
Compute variance of estimation error VAR(N,NSAM)
ISTAT= 0
no
Compute off-diagonal covariance of estimation error terms VAR(I,J) I≠J
0


0
IPRT= 0
yes
no
IG= 0
no
yes
K, GKS
PKKS
K,I,XM,ERR,
VAR(I,I,K)
ISTAT= 0
no
yes
VAR(I,J,K)
I≠J
P


P
IPLT= 0
no
STOP
yes
IGPLT= 1
yes
CALL PLOTP
GKS vs K
no
ITHVPL=1
yes
CALL PLOTP
PKKS vs K
no
IMTPLT= 1
yes
CALL PLOTP
XM vs K
no
Q


Q
ISMPLT= 1
no
yes
CALL PLOTP
ERR vs K
ISVPLT= 1
no
yes
CALL PLOTP
VAR vs K
STOP

A.4 Alphabetical Listing of Key Variables Used in the Monte-Carlo Program

AMISS(NSAM) - sample prediction miss distance vector

COVW(M,M) - covariance of forcing inputs matrix (double precision)

EI(N,N) - identity matrix (double precision)

ERR(N,NSAM) - sample estimation error vector

G(N,M) - the sample filter gain matrix (double precision)

GAMMA(N,N) - the matrix which relates the forcing inputs to the state vector (double precision)

GAMMAS(N,N) - the same as GAMMA but single precision

GK(N,M) - the sample filter gain matrix (single precision)

GKS(N,M,NSAM) - the filter gain schedule

H(M,N) - the observation matrix (double precision)

HS(M,N) - the observation matrix (single precision)

ITER - the ensemble member index

K - the sample index

PHI(N,N) - the state transition matrix (double precision)

PHIS(N,N) - the state transition matrix (single precision)

PKK(N,N) - the covariance of estimation error matrix (double precision)

PKKM1(N,N) - the one step prediction error covariance matrix (double precision)

PKKS(N,N) - the covariance of estimation error matrix (single precision)

Q(N,N) - the state excitation covariance matrix (double-precision)

R(N,N)	-	the covariance of measurement noise matrix (double precision)
SIGV(N)	-	the measurement noise standard deviation vector
SIGW(N)	-	the random forcing input standard deviation vector
SIGXZ(N)	-	the initial sample point standard deviation
V(N)	-	the measurement noise vector
W(N)	-	the forcing input vector
X(N)	-	the state vector
XHKK(N)	-	the state estimate vector
XHKKM1(N)	-	the one step state prediction vector
XM(N,NSAM)	-	the mean state trajectory over the ensemble
XS(N,NSAM)	-	the state trajectory for the current member of the ensemble
XZMEAN(N)	-	the mean of the initial state sample
Z(N)	-	the measurement vector

A.5 A List of IBM System/360 Source Library Subroutines Used by the Monte-Carlo Program

GAUSS3	-	matrix inversion by the method of Gauss
OVFLOW	-	utility routine required by SNORM
PLOTP	-	plot on printer-multiple curves
SNORM	-	normally distributed pseudo-random deviations (shuffled)

APPENDIX B

PREDICTION ACCURACY CALCULATION

The accuracy of the predictions which can be made from each sample estimate are of prime concern in a gun fire control system since the gun is aimed at a predicted target position. A relatively simple procedure for measuring the prediction accuracy over realistic time intervals is described below.

The time interval over which the predictions are made is based on a straight line approximation of the time of flight (TOF) versus range curve for a 5"/54 caliber projectile [6] as indicated in Fig. 4.

Measurement of the slope of the straight line approximation in Fig. 4 shows that

$$\text{TOF} \simeq 1.43 \times 10^{-3} \quad \text{(s/yd)} \times \text{Range (yds)} \qquad (1)$$

Taking the maximum effective range of the gun against an air target into consideration, the time interval over which predictions are made is limited to a maximum of ten seconds. This corresponds to a maximum effective range of approximately 7,000 yards.

A further approximation is made in order to simplify the matrix manipulations required. The TOF is truncated to an integer multiple of the sample period. The prediction is based on the equation

$$\underset{\sim}{\hat{x}}(k/k-N) = \underset{\sim}{\Phi}^{N}\underset{\sim}{\hat{x}}(k-N/k-N) \qquad (2)$$

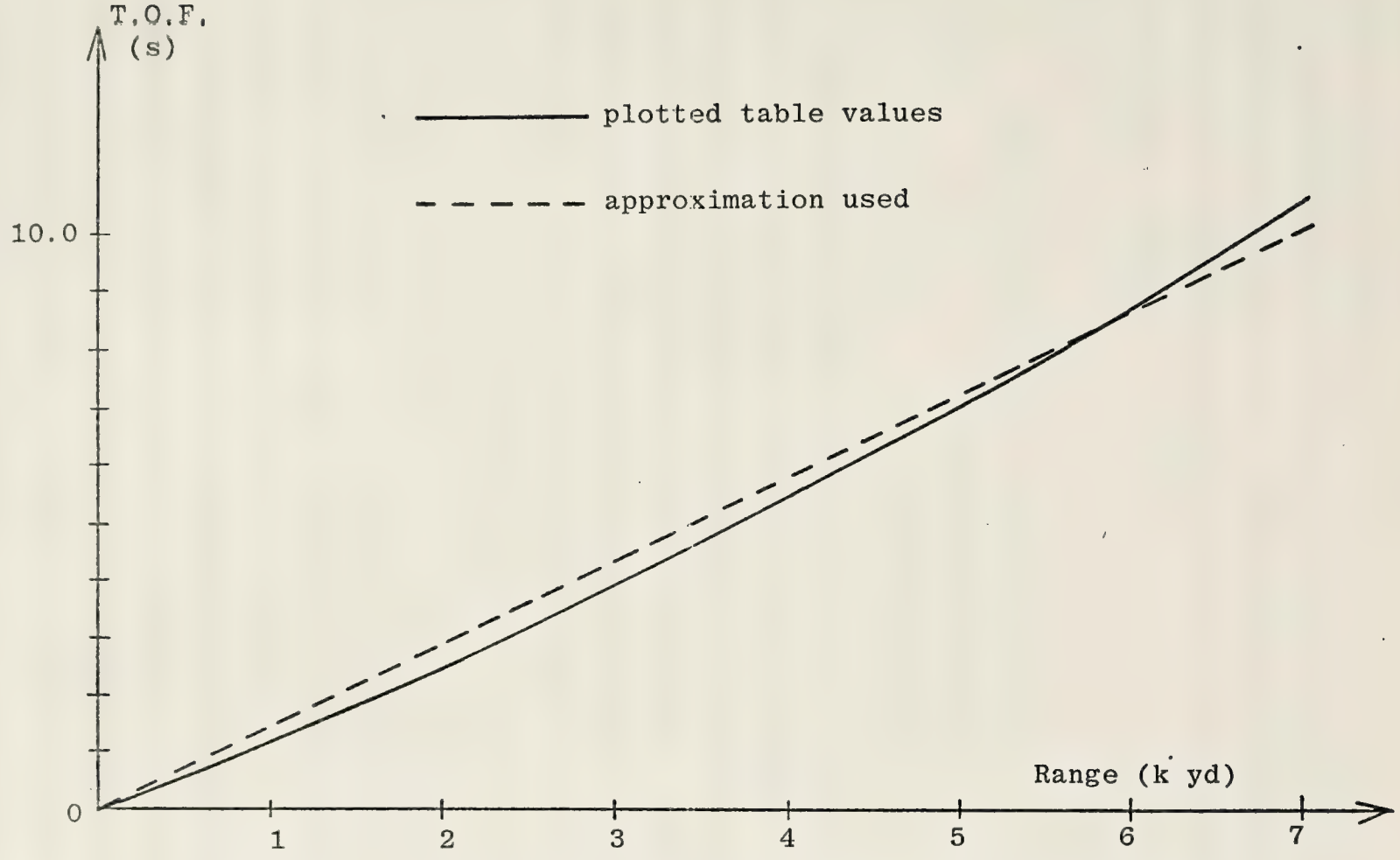


FIGURE 4 PLOT OF TIME OF FLIGHT VS. RANGE FOR A 5"/54 CAL. PROJECTILE

where N is the integer number of sample periods over which the prediction is made.

The prediction miss distance is calculated by finding the square root of the sum of the squares of the difference between the predicted position states and the respective true position states.

i.e.

$$(\text{miss distance})^2 = [\hat{x}_1(k/k-N)-x_1(k)]^2 + [\hat{x}_2(k/k-N)-x_2(k)]^2 + [\hat{x}_3(k/k-N)-x_3(k)]^2 \qquad (3)$$

The subroutine which calculates the prediction miss distance in the Monte-Carlo program stores a running sum of miss distances for each sample point over the ensemble so that the mean miss distance can be calculated. The subroutine also counts the number of predictions made (the number of samples which meet the projectile firing criteria), the number of predictions with miss distance less than 24 feet and the number of predictions with miss distances less than 48 feet for each run over the ensemble. The number of predictions with miss distance less than 48 feet at each sample point over the ensemble is also recorded.

The Fortran listing of subroutine Burst is included as part of the Monte-Carlo program in Appendix A. Notice that equation (2) is not evaluated in the Computer by raising the n×n matrix $\underset{\sim}{\Phi}$ to the N^{th} power by direct matrix algebra, which would be very time consuming, but is calculated by

operating on the individual components of $\underset{\sim}{\Phi}$. The N-step state transition matrix is given by

$$\underset{\sim}{\Phi}^N = \begin{bmatrix} \underset{\sim}{\Phi}_i & \underset{\sim}{0} & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{\Phi}_i & \underset{\sim}{0} \\ \underset{\sim}{0} & \underset{\sim}{0} & \underset{\sim}{\Phi}_i \end{bmatrix} \tag{4}$$

where for the constant-velocity model

$$\underset{\sim}{\Phi}_i = \begin{bmatrix} 1.0 & NT \\ 0.0 & 1.0 \end{bmatrix} \tag{5}$$

and for the correlated random acceleration model

$$\underset{\sim}{\Phi}_i = \begin{bmatrix} 1.0 & NT & \underset{\sim}{\Phi}_{13} \sum_{n=1}^{N} \underset{\sim}{\Phi}_{33}^{n-1} + T\underset{\sim}{\Phi}_{23} \sum_{n=1}^{N-1} [N-n]\underset{\sim}{\Phi}_{33}^{n-1} \\ 0.0 & 1.0 & \underset{\sim}{\Phi}_{23} \sum_{n=1}^{N} \underset{\sim}{\Phi}_{33}^{n-1} \\ 0.0 & 0.0 & \underset{\sim}{\Phi}_{33}^{N} \end{bmatrix} \tag{6}$$

where

$$\underset{\sim}{\Phi}_{13} = \frac{T}{\alpha} - \frac{1}{\alpha^2}\left[1-e^{-\alpha T}\right] \tag{7}$$

$$\underset{\sim}{\Phi}_{23} = \frac{1}{\alpha}\left[1-e^{-\alpha T}\right] \tag{8}$$

$$\underset{\sim}{\Phi}_{33} = e^{-\alpha T} \tag{9}$$

When the prediction is to be based on an assumption of constant velocity and the estimator is based on the model which assumes correlated random accelerations, $\underset{\sim}{\Phi}^N$ is calculated from equation (6) by setting $\underset{\sim}{\Phi}_{13}$, $\underset{\sim}{\Phi}_{23}$, and $\underset{\sim}{\Phi}_{33}$ equal to zero.

Equations (5) and (6) are derived by self-multiplying the matrices of equations (3.1) and (3.17) respectively and establishing a pattern.

APPENDIX C

A TARGET TRAJECTORY GENERATING PROGRAM

C.1 Description of the Program

In order to test the filter's performance against maneuvering targets it is desirable to have a realistic trajectory for which position, velocity and acceleration states are known at each sample point.

The maneuver decided upon is a periodic, planar maneuver described by the equations

$$y(x,t) = pA\sin^2(kx), \tag{1}$$

where

$$p = (-1)^n, \qquad n = \begin{cases} 0, & kx < \pi \\ 1, & kx \geq \pi, \end{cases}$$

and

$$k = \frac{2\pi}{\left[\begin{array}{l}\text{the distance traveled along the x-axis} \\ \text{during the maneuver}\end{array}\right]}$$

$$\dot{y}(x,\dot{x},t) = 2pAk\dot{x}\cos(kx)\sin(kx) \tag{2}$$

and

$$\ddot{y}(x,\dot{x},\ddot{x},t) = 2pAk\{\ddot{x}\cos(kx)\sin(kx) + k\dot{x}^2[2\cos^2(kx)-1]\} \tag{3}$$

The form of the maneuver is shown in Fig. 5. The variables x, $\dot{x}$, and $\ddot{x}$ have been used to represent $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$ respectively in equations (1) through (14) to make the notation less awkward.

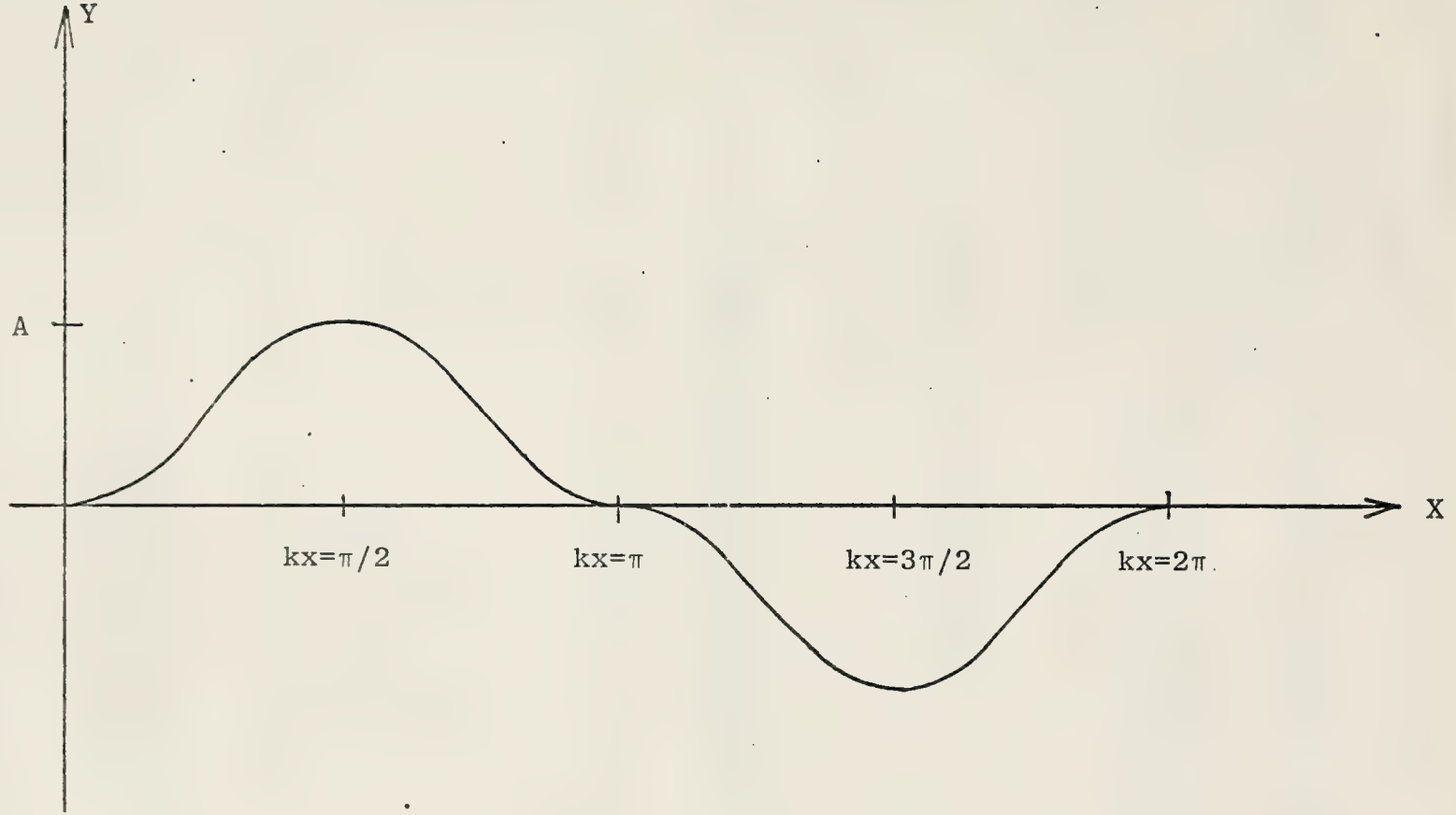


FIGURE 5 THE PLANAR MANEUVER

The two dimensional geometry of the maneuver yields the relationship

$$v^2 = \dot{x}^2 + \dot{y}^2 \tag{4}$$

where v, the speed of the aircraft tangent to the x-y plane, is assumed to be constant.

Transposing the terms of equation (4), substituting the right side of equation (2) for $\dot{y}$, and taking the square root of both sides yields

$$\dot{x} = \frac{v}{[1+4A^2k^2\cos^2(kx)\sin^2(kx)]^{\frac{1}{2}}} . \tag{5}$$

Equation (5) is differentiated with respect to time with the result

$$\ddot{x} = \frac{4A^2k^2v\dot{x}\sin(kx)\cos(kx)[1-2\cos^2(kx)]}{[1+4A^2k^2\cos^2(kx)\sin^2(kx)]^{3/2}} \tag{6}$$

If the system states are defined as

$$x_1 = x(t) \tag{7}$$

$$x_2 = \dot{x}(t) \tag{8}$$

$$x_3 = y(x,t) \tag{9}$$

$$x_4 = \dot{y}(x,\dot{x},t) \tag{10}$$

the resulting nonlinear state equations are

$$\dot{x}_1 = x_2 \tag{11}$$

$$\dot{x}_2 = \frac{4A^2k^2vx_2\sin(kx_1)\cos(kx_1)[1-2\cos^2(kx_1)]}{[1+4A^2k^2\cos^2(kx_1)\sin^2(kx_1)]^{3/2}} \tag{12}$$

$$\dot{x}_3 = x_4 \tag{13}$$

$$\dot{x}_4 = 2pAk\{\dot{x}_2\cos(kx_1)\sin(kx_1)-kx_2^2[1-2\cos^2(kx_1)]\} \tag{14}$$

The state equations are used in a computer program to make up the maneuvering portion of the trajectory. The computer program utilizes subroutine RKLDEQ, a Runge-Kutta-Gill differential equation solver, which is a member of the IBM System 360 Source Library.

In order that filter's performance against a target which maneuvers after the filter gains have reached steady state can be studied, provision is made for a constant-velocity section of target trajectory preceding the maneuver. A constant-velocity section of track following the maneuvers allows the filter's recovery time to be examined.

Although the trajectory is originally generated with the x-axis as its axis of symmetry, it may be rotated to any bearing angle and any first quadrant elevation angle. The geometry involved in the rotation of the axis of symmetry is illustrated in Fig. 6 through Fig. 8. As indicated in Fig. 6 for the elevation angle rotation, a new coordinate system is established which places the axis of symmetry, now labeled the x'-axis, at the desired elevation angle, E. The x and z-coordinate states can be calculated from the x' coordinate states.

$$x(k) = x'(k)\cos(E) \tag{15}$$

$$\dot{x}(k) = \dot{x}'(k)\cos(E) \tag{16}$$

$$\ddot{x}(k) = \ddot{x}'(k)\cos(E) \tag{17}$$

$$z(k) = x'(k)\sin(E) \tag{18}$$

$$\dot{z}(k) = \dot{x}'(k)\sin(E) \tag{19}$$

$$\ddot{z}(k) = \ddot{x}'(k)\sin(E) \tag{20}$$

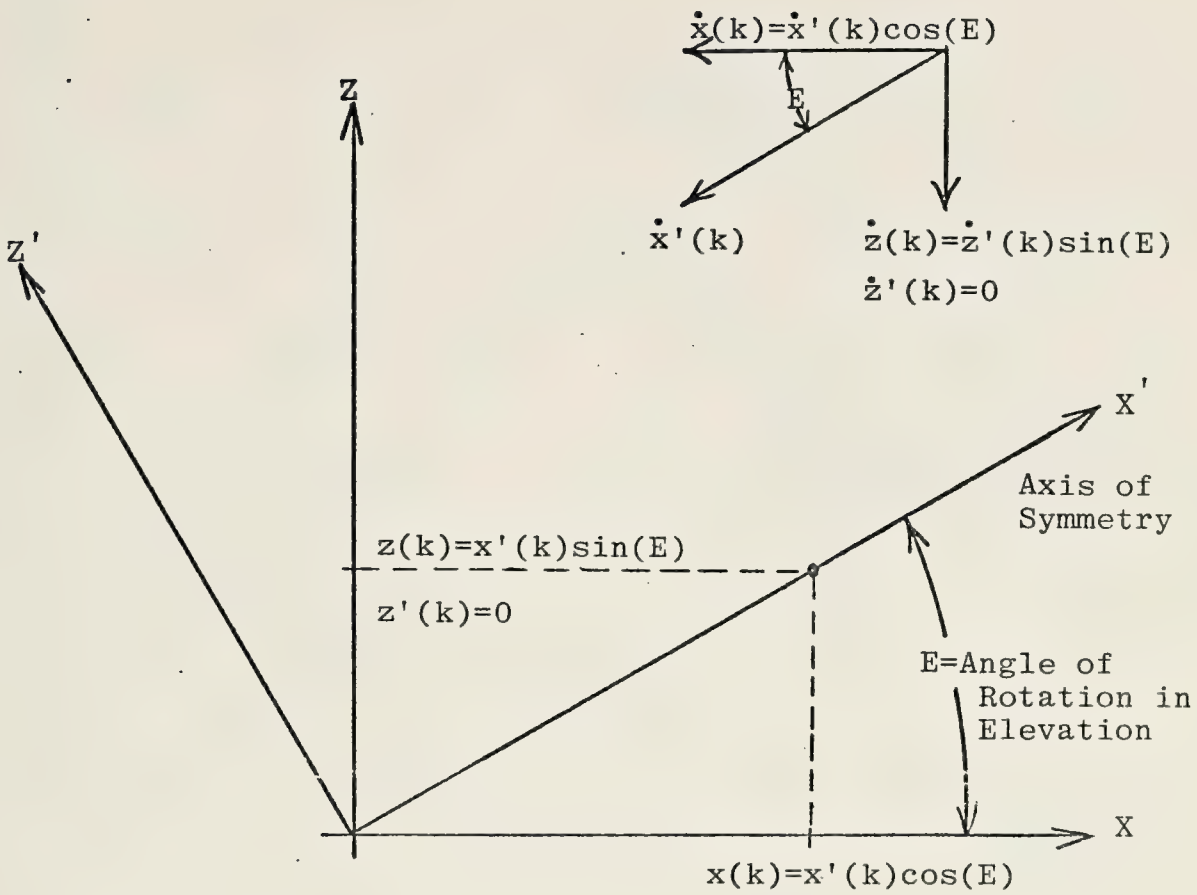


FIGURE 6 ROTATION OF THE TRACK IN ELEVATION (Vertical Plane)

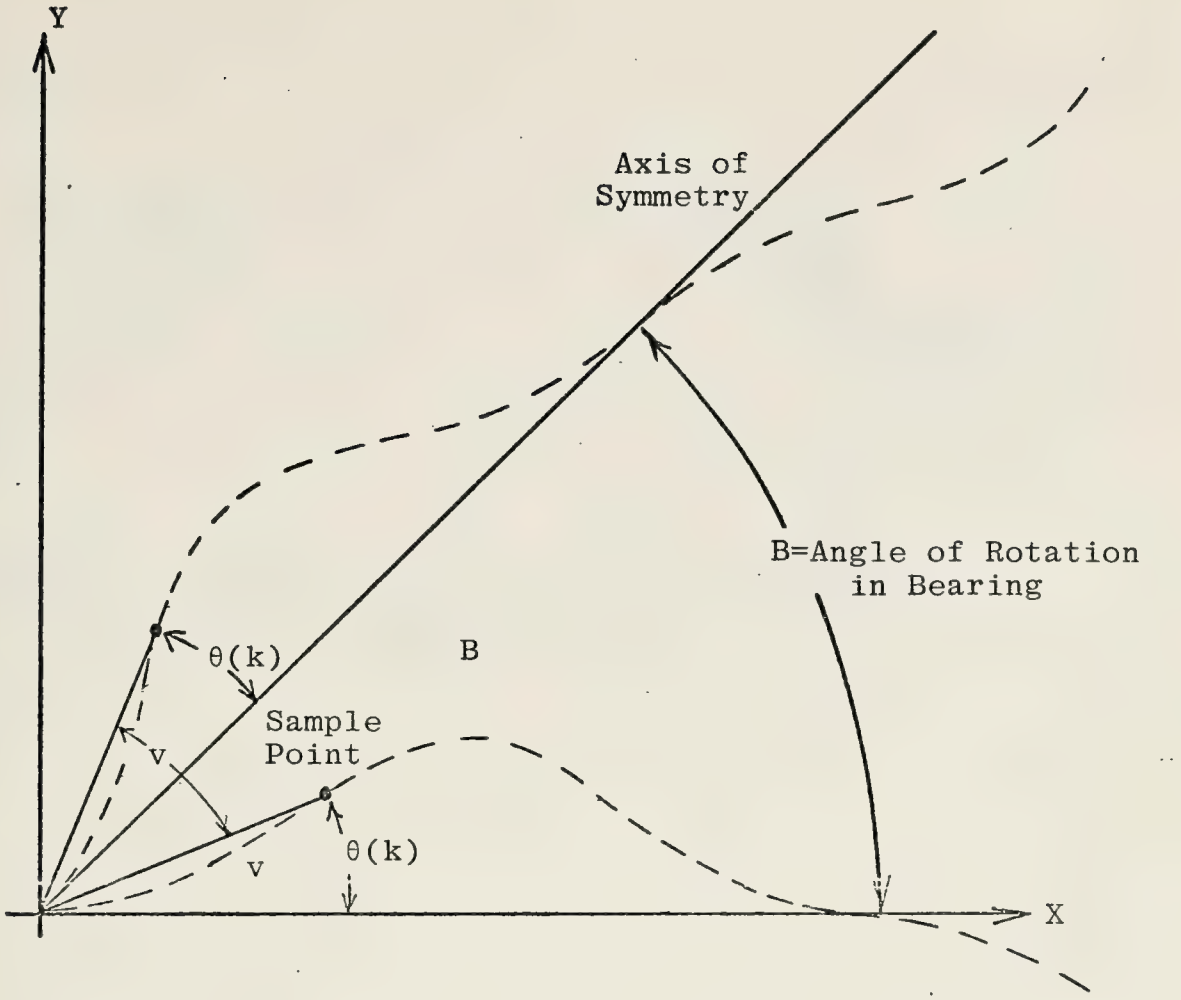


FIGURE 7 ROTATION OF THE TRACK AXIS IN BEARING

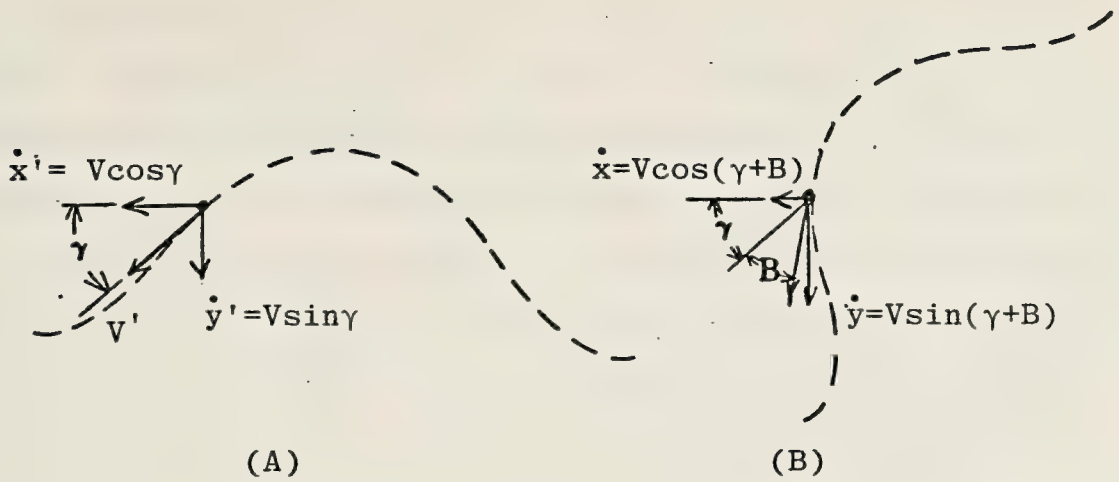


FIGURE 8 VELOCITY VECTOR BEFORE (A) AND AFTER (B) ROTATION THROUGH ANGLE B

The y-coordinate states are not affected by the elevation angle rotation since the rotation is about the y-axis. The bearing angle rotation is depicted in Fig. 7 -- the case shown is for a zero elevation angle. The axis of symmetry is rotated through the angle B, which results in a point on the trajectory curve being displaced from the x-axis by a total angle of B + θ. Figure 7 shows the relationship between the x- and y-direction velocities before and after rotation. The z-coordinate states are not affected by the bearing rotation since the rotation is about the z-axis. The values of the x and y-coordinate states after rotation of the trajectory's axis of symmetry through elevation angle E and bearing angle B are given by

$$x(k) = R(k)\cos[B+\theta(k)]\cos(E) , \tag{21}$$

$$\dot{x}(k) = [\dot{x}'(k)\cos(B)-\dot{y}'(k)\sin(B)]\cos(E) , \tag{22}$$

$$\ddot{x}(k) = [\ddot{x}'(k)\cos(B)-\ddot{y}'(k)\sin(B)]\cos(E) , \tag{23}$$

$$y(k) = R(k)\sin[B+\theta(k)]\cos(E) , \tag{24}$$

$$\dot{y}(k) = \dot{x}'(k)\cos E\sin B+\dot{y}'(k)\cos B , \tag{25}$$

$$\ddot{y}(k) = \ddot{x}'(k)\cos E\sin B+\ddot{y}'\cos B , \tag{26}$$

where

$$R(k) = \sqrt{x'(k)^2+y'(k)^2} = \sqrt{x(k)^2+y(k)^2+z(k)^2} \tag{27}$$

and

$$\theta(k) = \arctan [y'(k)/x'(k)] \tag{28}$$

Equations (22) and (25) are derived from the trigonometric relationship shown in Fig. 7. Equations (23) and (26) are derived from a similar relationship between the acceleration states.

Provision is made for the trajectory to have non-zero intercepts with the y and/or z axis. This is accomplished simply by adding any desired value to the y-and/or z-position states at each sample point.

A subroutine is provided which performs conversion of all state values from cartesian to spherical coordinate quantities for cases in which spherical coordinate data is required. The spherical coordinate data is in units of yards in range and radians in bearing and elevation.

C.2 Relationship Between Velocity, Period of Maneuver, Amplitude of Maneuver, and Acceleration

The magnitude of the total acceleration for the x-y plane maneuver is given by

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} \tag{29}$$

where $\ddot{x}$ and $\ddot{y}$ are given by equations (6) and (3) respectively. Making the substitutions as indicated in equation (29) and laboriously solving for a yields

$$a = \frac{2Ak^2v^2[1-2\cos^2(k)]}{1+4A^2k^2\cos^2(kx)\sin^2(kx)} \tag{30}$$

which can be further simplified by utilizing the trigonometric relationship

$$\cos 2\alpha = 2\cos^2\alpha - 1 \tag{31}$$

and

$$\sin 2\alpha = 2\sin\alpha\cos\alpha \tag{32}$$

with the result

$$a = \frac{-2Ak^2v^2\cos(2kx)}{1 + A^2k^2\sin^2(2kx)} \tag{33}$$

To find the point, with respect to x-position, at which the maximum acceleration occurs during the maneuver it is necessary to differentiate equation (33) with respect to x and set the result equal to zero. This yields

$$4A^2k^2v^2\sin(2kx)[1+4Ak^2\sin^2(2kx)+Ak^2\cos^2(2kx)] = 0 \tag{34}$$

which is true when

$$\sin(2kx) = 0,$$

$$\text{i.e. } 2kx=n\pi, \quad n=0,1,2,....$$

Substitution of $2kx = n\pi$ into equation (28) yields

$$|a|_{max} = 2Ak^2v^2 \tag{35}$$

Equation (35) gives the relationship between target speed, maneuver amplitude, maneuver period, and the maximum acceleration required to complete the maneuver. This relationship is useful as an aid in choosing values for the variables involved.

C.3 Examples of Track Generating Program Use

Example 1. A trajectory is to be generated which approaches the origin from a range of 3,500 yards with a 20 degree dive angle at a constant speed of 200 yd/s. A 2.5 g maneuver of approximately ten seconds duration is to commence when the target is at a range of 2,700 yards. Sixty samples are to be generated at a four-hertz sampling rate.

The first task is to convert the track specifications into the data required by the track generating program.

A maneuver duration of ten seconds is specified. To calculate the distance traveled along the axis of symmetry during a 2.5 g, ten second; $\sin^2$ maneuver would not be a simple task. However experience has shown that the change in range rate during such a maneuver is relatively small, therefore the distance is approximately

$$x_{per} = (10s)(200yd/s) = 2000 \text{ yd.} \tag{37}$$

Equation (36) can now be used to solve for the amplitude of the maneuver.

$$A = \frac{2.5(g) \times 10.73(yd/s.g)}{2[\frac{2}{2000(yd)}]^2[200(yd/s)]^2} \tag{38}$$

$$= 33.97 \text{ yd.}$$

The four hertz sampling rate inverts to a 0.25 s sampling period.

The data is now available in the proper form to make up the data input cards for the track generating program. The cards must contain the following information.

Card 1:

S(1,1) = initial range = 3500.0
S(2,1) = initial range rate = -200.0

Card 2:

RATE	= sampling period	= 0.25
V	= target speed	= 200.0
A	= maneuver amplitude	= 33.97
XPER	= maneuver duration	= 2000.0
BR	= bearing angle	= 0.0 (no bearing specified)
E	= elevation angle	= 20.0
RMAN	= maneuver commencement range	= 2700.0
TRKEND	= trajectory end range (the track is to be ended by the sample count)	=-9999.9

Card 3:

CPA	= y-direction displacement	= 0.0
HO	= z-direction displacement	= 0.0

Card 4:

NSAM	= no. of samples desired	= 60
N	= no. of states generated	= 9
ICOORD	= coordinate system indicator (data is to be in cartesian coordinates)	= 0
IPUNCH	= card output indicator (the output data is to be punched onto cards)	= 1

Table VIII lists the data generated by the track generating program for this example.

Example 2. A level, constant-velocity trajectory beginning at the point x=5000 yards, y=1000 yards and z=333.3 yards in cartesian coordinates and proceeding in the negative x-direction with a speed of 200 yds/s until the x-direction displacement is 2800 yards is to be produced. The sampling period is 0.25 seconds. The output data is to be in spherical coordinates.

This trajectory can be generated quite simply by utilizing the CPA and HO features of the program. Since the trajectory is generated initially with the x-axis as its

TABLE VIII TRACK GENERATING PROGRAM OUTPUT FOR EXAMPLE 1

X-POSITION	X-VELOCITY	X-ACCELER	Y-POSITION	Y-VELOCITY	Y-ACCELER	Z-POSITION	Z-VELOCITY	Z-ACCELER	
3285.	-187.9	C.0	0.0	0.0	0.0	1157.	-68.40	0.0	1
3442.	-187.9	C.0	0.0	0.0	0.0	1180.	-68.40	0.0	2
3195.	-187.9	C.0	0.0	0.0	0.0	1163.	-68.40	0.0	3
3148.	-187.9	0.0	0.0	0.0	0.0	1146.	-68.40	0.0	4
3101.	-187.9	C.0	0.0	0.0	0.0	1129.	-68.40	0.0	5
3054.	-187.9	C.0	0.0	0.0	0.0	1112.	-68.40	0.0	6
3007.	-187.9	0.0	0.0	0.0	0.0	1094.	-68.40	0.0	7
2960.	-187.9	0.0	0.0	0.0	0.0	1077.	-68.40	0.0	8
2913.	-187.9	C.0	0.0	0.0	0.0	1060.	-68.40	0.0	9
2866.	-187.9	0.0	0.0	0.0	0.0	1043.	-68.40	0.0	10
2819.	-187.9	0.0	0.0	0.0	0.0	1026.	-68.40	0.0	11
2772.	-187.9	C.0	0.0	0.0	0.0	1009.	-68.40	0.0	12
2725.	-187.9	C.0	0.0	0.0	0.0	991.9	-68.40	0.0	13
2678.	-187.9	C.0	0.0	0.0	0.0	974.8	-68.40	0.0	14
2631.	-187.9	C.0	0.0	0.0	0.0	957.7	-68.40	0.0	15
2584.	-187.9	0.0	0.0	0.0	0.0	940.6	-68.40	0.0	16
2537.	-187.9	C.0	0.0	0.0	0.0	923.5	-68.40	0.0	17
2490.	-187.8	C.7887	-0.7599	-6.501	-25.45	906.4	-68.37	0.2871	18
2443.	-187.6	1.269	-3.002	-12.42	-21.54	899.3	-68.27	0.4618	19
2396.	-187.2	1.261	-6.499	-17.10	-15.56	872.2	-68.15	0.4591	20
2350.	-187.0	C.7803	-10.90	-20.09	-8.188	855.2	-68.06	0.2840	21
2303.	-186.9	C.1167D-01	-15.78	-21.13	-0.1164	838.2	-68.02	0.4247D-02	22
2256.	-187.0	-C.7812	-20.67	-20.15	7.965	821.2	-68.05	-0.2771	23
2209.	-187.2	-1.254	-25.10	-17.21	15.37	804.2	-68.15	-0.4563	24
2163.	-187.6	-1.276	-28.63	-12.58	21.39	787.1	-68.27	-0.4643	25
2116.	-187.8	-0.8078	-30.91	-6.682	25.38	770.0	-68.36	-0.2940	26
2069.	-187.9	-0.2401D-01	-31.71	-0.1011	26.82	752.9	-68.40	-0.4738D-02	27
2022.	-187.8	C.7693	-30.95	6.499	25.53	735.8	-68.37	0.2800	28
1975.	-187.6	1.261	-28.71	12.45	21.68	718.8	-68.27	0.4591	29
1928.	-187.3	1.269	-25.20	17.17	15.75	701.7	-68.15	0.4617	30
1881.	-187.0	C.7991	-20.77	20.21	8.411	684.7	-68.06	0.2909	31
1834.	-186.9	C.3501D-01	-15.86	21.31	0.3491	667.7	-68.02	0.4274D-01	32
1788.	-187.0	-0.7419	-10.92	20.38	-7.742	650.7	-68.05	-0.2700	33
1741.	-187.2	-1.246	-6.436	17.49	-15.17	633.6	-68.14	-0.4534	34
1694.	-187.6	-1.282	-2.835	12.91	-21.25	616.6	-68.26	-0.4667	35
1647.	-187.8	-C.8267	-0.4722	7.041	-25.30	599.5	-68.36	-0.3009	36
1600.	-187.9	-C.4801D-01	0.4176	0.4705	-26.82	582.4	-68.40	-0.1747D-01	37
1553.	-187.8	0.7496	1.167	6.454	25.60	565.3	-68.37	0.2728	38
1506.	-187.6	1.254	3.404	12.43	21.82	548.2	-68.28	0.4563	39
1459.	-187.3	1.275	6.912	17.19	15.95	531.2	-68.16	0.4642	40
1413.	-187.0	C.8177	11.35	20.28	8.633	514.1	-68.06	0.2976	41
1366.	-186.9	C.5833D-01	16.29	21.44	0.5818	497.1	-68.02	0.2123D-01	42
1319.	-187.0	-0.7224	21.26	20.57	-7.518	480.1	-68.05	-0.2629	43
1272.	-187.2	-1.237	25.80	17.74	-14.98	463.1	-68.14	-0.4503	44
1226.	-187.5	-1.288	29.46	13.19	-21.10	446.1	-68.26	-0.4689	45
1179.	-187.8	-C.8453	31.90	7.357	-25.21	429.0	-68.36	-0.3077	46
1132.	-187.9	-C.7200D-01	32.86	0.7972	-26.81	411.9	-68.40	-0.2620D-01	47
1085.	-187.9	0.7297	32.27	-5.820	-25.68	394.8	-68.37	0.2656	48
1038.	-187.6	1.246	30.18	-11.82	-21.96	377.7	-68.28	0.4533	49
990.9	-187.3	1.282	26.81	-16.62	-16.14	360.6	-68.16	0.4666	50
944.1	-187.0	0.8360	22.50	-19.77	-8.854	343.6	-68.06	0.3043	51
897.4	-186.9	C.8164D-01	17.67	-20.99	-0.8145	326.6	-68.02	0.2971D-01	52
850.6	-187.0	-C.7026	12.80	-20.17	7.293	309.6	-68.05	-0.2557	53
803.9	-187.2	-1.228	8.347	-17.39	14.78	292.6	-68.14	-0.4471	54
757.0	-187.5	-1.294	4.759	-12.89	20.95	275.5	-68.26	-0.4710	55
710.1	-187.8	-0.8637	2.392	-7.084	25.13	258.5	-68.36	-0.3143	56
663.1	-187.9	-C.9596D-01	1.489	-0.5363	26.80	241.4	-68.40	-0.3493D-01	57
616.1	-187.9	C.7095	2.146	6.089	25.74	224.3	-68.37	0.2583	58
569.2	-187.9	C.0	0.0	0.0	0.0	207.2	-68.40	0.0	59
522.2	-187.9	0.0	0.0	0.0	0.0	190.1	-68.40	0.0	60

axis of symmetry and the y and z-direction displacements are added afterward, the specified values of initial x-position and velocity are taken as the initial range and range-rate respectively. The other required input values are given directly in the track specifications.

The input data cards contain the following information.

Card 1:

S(1,1) = initial range (x-position) = 5000.0
S(2,1) = initial range rate (x-velocity) = -200.0

Card 2:

RATE	= sampling period	= 0.25
V	= target speed	= 200.0
A	= maneuver amplitude	= 0.0
XPER	= maneuver duration	= 0.0
BR	= bearing angle	= 0.0
E	= elevation angle	= 0.0
RMAN	= maneuver commencement range	= 0.0
TRKEND	= trajectory end range	= 2800.0

Card 3:

CPA	= y-direction displacement	= 1000.0
HO	= z-direction displacement	= 333.3

Card 4:

NSAM	= number of samples (the track is range limited)	= 200
N	= number of states generated	= 9
ICOORD	= coordinate system indicator (spherical coordinate data desired)	= 1
IPUNCH	= card output indicator (no cards are desired)	= 0

Table IX lists the trajectory data generated by the track generating program for this example.

TABLE IX TRACK GENERATING PROGRAM OUTPUT FOR EXAMPLE 2

OUTPUT DATA IN SPHERICAL COORDINATES

RANGE	RDOT	R2DOT	BEARING	BDOT	B2DOT	ELEVATION	EDOT	E2DOT	
5110.	-195.7	0.3331	0.1974	0.7892D-02	0.5917D-03	0.6528D-01	0.2504D-02	0.4703	1
5061.	-195.6	0.3429	0.1993	0.7842D-02	0.6089D-03	0.6591D-01	0.2551D-02	0.4786	2
5012.	-195.5	0.3530	0.2013	0.7997D-02	0.6267D-03	0.6655D-01	0.2600D-02	0.4872	3
4963.	-195.4	0.3635	0.2033	0.8156D-02	0.6452D-03	0.6721D-01	0.2651D-02	0.4959	4
4914.	-195.3	0.3745	0.2054	0.8319D-02	0.6644D-03	0.6788D-01	0.2702D-02	0.5049	5
4866.	-195.3	0.3859	0.2075	0.8488D-02	0.6844D-03	0.6856D-01	0.2756D-02	0.5141	6
4817.	-195.2	0.3977	0.2096	0.8662D-02	0.7052D-03	0.6926D-01	0.2811D-02	0.5235	7
4768.	-195.1	0.4100	0.2118	0.8841D-02	0.7269D-03	0.6997D-01	0.2867D-02	0.5332	8
4719.	-194.9	0.4229	0.2141	0.9025D-02	0.7494D-03	0.7069D-01	0.2925D-02	0.5432	9
4671.	-194.8	0.4362	0.2163	0.9216D-02	0.7728D-03	0.7143D-01	0.2985D-02	0.5534	10
4622.	-194.7	0.4502	0.2187	0.9412D-02	0.7972D-03	0.7218D-01	0.3047D-02	0.5638	11
4573.	-194.6	0.4647	0.2210	0.9614D-02	0.8227D-03	0.7295D-01	0.3110D-02	0.5746	12
4525.	-194.5	0.4798	0.2235	0.9823D-02	0.8492D-03	0.7374D-01	0.3176D-02	0.5856	13
4476.	-194.4	0.4957	0.2260	0.1004D-01	0.8768D-03	0.7454D-01	0.3243D-02	0.5969	14
4427.	-194.2	0.5121	0.2285	0.1026D-01	0.9056D-03	0.7536D-01	0.3313D-02	0.6085	15
4379.	-194.1	0.5294	0.2311	0.1049D-01	0.9357D-03	0.7620D-01	0.3385D-02	0.6204	16
4330.	-194.0	0.5474	0.2337	0.1073D-01	0.9670D-03	0.7705D-01	0.3459D-02	0.6327	17
4282.	-193.8	0.5662	0.2365	0.1098D-01	0.9998D-03	0.7793D-01	0.3535D-02	0.6452	18
4233.	-193.7	0.5858	0.2392	0.1123D-01	0.1034D-02	0.7882D-01	0.3614D-02	0.6582	19
4185.	-193.6	0.6064	0.2421	0.1149D-01	0.1070D-02	0.7974D-01	0.3696D-02	0.6714	20
4137.	-193.4	0.6279	0.2450	0.1176D-01	0.1107D-02	0.8067D-01	0.3780D-02	0.6850	21
4088.	-193.2	0.6504	0.2480	0.1205D-01	0.1146D-02	0.8163D-01	0.3867D-02	0.6990	22
4040.	-193.1	0.6741	0.2510	0.1234D-01	0.1187D-02	0.8260D-01	0.3957D-02	0.7134	23
3992.	-192.9	0.6988	0.2541	0.1264D-01	0.1230D-02	0.8360D-01	0.4050D-02	0.7281	24
3943.	-192.7	0.7247	0.2573	0.1295D-01	0.1275D-02	0.8463D-01	0.4146D-02	0.7433	25
3895.	-192.5	0.7519	0.2606	0.1328D-01	0.1322D-02	0.8568D-01	0.4245D-02	0.7588	26
3847.	-192.3	0.7805	0.2640	0.1361D-01	0.1372D-02	0.8675D-01	0.4348D-02	0.7748	27
3799.	-192.1	0.8105	0.2674	0.1396D-01	0.1423D-02	0.8785D-01	0.4455D-02	0.7912	28
3751.	-191.9	0.8420	0.2709	0.1433D-01	0.1478D-02	0.8898D-01	0.4565D-02	0.8081	29
3703.	-191.7	0.8752	0.2746	0.1470D-01	0.1535D-02	0.9013D-01	0.4679D-02	0.8254	30
3655.	-191.5	0.9100	0.2783	0.1509D-01	0.1595D-02	0.9132D-01	0.4798D-02	0.8431	31
3607.	-191.3	0.9467	0.2821	0.1550D-01	0.1658D-02	0.9253D-01	0.4920D-02	0.8614	32
3560.	-191.0	0.9854	0.2861	0.1592D-01	0.1724D-02	0.9378D-01	0.5048D-02	0.8801	33
3512.	-190.8	1.026	0.2901	0.1636D-01	0.1794D-02	0.9506D-01	0.5179D-02	0.8993	34
3464.	-190.5	1.069	0.2942	0.1682D-01	0.1867D-02	0.9637D-01	0.5316D-02	0.9190	35
3417.	-190.2	1.114	0.2985	0.1730D-01	0.1945D-02	0.9772D-01	0.5458D-02	0.9392	36
3369.	-190.0	1.162	0.3029	0.1779D-01	0.2026D-02	0.9910D-01	0.5606D-02	0.9599	37
3322.	-189.7	1.213	0.3074	0.1831D-01	0.2112D-02	0.1005	0.5759D-02	0.9811	38
3274.	-189.4	1.266	0.3120	0.1885D-01	0.2203D-02	0.1020	0.5918D-02	1.003	39
3227.	-189.0	1.323	0.3168	0.1941D-01	0.2299D-02	0.1035	0.6083D-02	1.025	40
3180.	-188.7	1.382	0.3218	0.2000D-01	0.2400D-02	0.1050	0.6255D-02	1.048	41
3133.	-188.3	1.446	0.3268	0.2061D-01	0.2507D-02	0.1066	0.6434D-02	1.071	42
3086.	-188.0	1.513	0.3321	0.2125D-01	0.2620D-02	0.1082	0.6619D-02	1.095	43
3039.	-187.6	1.584	0.3375	0.2192D-01	0.2740D-02	0.1099	0.6813D-02	1.119	44
2992.	-187.2	1.660	0.3430	0.2262D-01	0.2866D-02	0.1116	0.7014D-02	1.143	45

C.4 The Track Generating Program Listing

```
C       THE FOLLOWING PROGRAM GENERATES A SIMULATED AIRCRAFT                    10
C       TRAJECTORY. THE TRAJECTORY MAY BE GENERATED WITH A MANEUVER IN          20
C       THE TRAVERSE PLANE BASED ON THE EQUATION     Y = A*SIN(X)**2            30
C       WHERE THE AMPLITUDE, PERIOD, AND COMMENCEMENT RANGE OF THE              40
C       MANEUVER MAY BE SELECTED BY THE USER. A MAXIMUM OF 200 SAMPLE           50
C       POINTS MAY BE GENERATED WITH THE PRESENT PROGRAM DIMENSIONS.            60
C                                                                               70
C       THE OUTPUT DATA MAY BE IN EITHER CARTESIAN OR SPHERICAL                 80
C       COORDINATES AS SELECTED BY THE USER.                                    90
C                                                                              100
C                                                                              110
      IMPLICIT REAL*8(A-H,Q-Z)                                                 120
      REAL *8PHI,P                                                             130
      DIMENSION S(9,200), TEMPL(200), PX(200), PXVEL(200), PXACC(200), P       140
     1Y(200), PYVEL(200), PYACC(200), PK(200), Y(4), F(4)                      150
C                                                                              160
C                                                                              170
C       THE FIRST DATA CARD PROVIDED MUST CONTAIN VALUES FOR THE                180
C       INITIAL CONDITIONS FOR THE RANGE AND RANGE RATE. THE FORMAT IS          190
C       (2F10.2)                                                               200
C                                                                              210
C                                                                              220
      READ (5,1) (S(I,1),I=1,6)                                                230
    1 FORMAT (8F10.2)                                                          240
C                                                                              250
C                                                                              260
C       THE SYSTEM STATES ARE DEFINED AS FOLLOWS.                              270
C                                                                              280
C         S(1,K)= X POSITION                                                   290
C         S(2,K)= X VELOCITY                                                   300
C         S(3,K)= X ACCELERATION                                               310
C         S(4,K)= Y POSITION                                                   320
C         S(5,K)= Y VELOCITY                                                   330
C         S(6,K)= Y ACCELERATION                                               340
C         S(7,K)= Z POSITION                                                   350
C         S(8,K)= Z VELOCITY                                                   360
C         S(9,K)= Z ACCELERATION                                               370
C                                                                              380
C       K = 1,2,3,.........,NSAM                                               390
C                                                                              400
C                                                                              410
C                                                                              420
C                                                                              430
C                                                                              440
C                                                                              450
C                                                                              460
C                                                                              470
C                                                                              480
```

```
      READ (5,1) RATE,V,A,XPER,BR,E,RMAN,TRKEND                          490
C                                                                        500
C        THE SECOND DATA CARD PROVIDED MUST CONTAIN VALUES FOR THE       510
C        FOLLOWING VARIABLES IN THE ORDER LISTED.. THE FORMAT IS 8F10.2. 520
C                                                                        530
C          RATE = SAMPLING PERIOD. (SEC.)                                540
C                                                                        550
C          V = SPEED OF AIRCRAFT (YARDS/SEC.).                           560
C                                                                        570
C          A = AMPLITUDE OF MANEUVER (YARDS). THE COEFFICIENT OF SIN**2  580
C          IN THE TRAVERSE PLANE MANEUVER.                               590
C                                                                        600
C          XPER = DURATION OF THE MANEUVER AS MEASURED IN RANGE (YARDS). 610
C                                                                        620
C          BR= THE BEARING ANGLE FROM WHICH THE TARGET IS TO APPROACH    630
C          THE ORIGIN. (DEGREES)  ONLY POSITIVE ANGLES MAY BE USED.      640
C                                                                        650
C          E = THE ELEVATION ANGLE FROM WHICH THE TARGET IS TO APPRCACH  660
C          THE ORIGIN. (DEGREES)  ONLY FIRST QUADRANT ANGLES MAY BE USED 670
C                                                                        680
C          RMAN = RANGE AT WHICH MANEUVER IS TO BEGIN. (YARDS)           690
C                                                                        700
C          TRKEND= RANGE AT WHICH THE TRACK IS TO END IF THE TRACK IS TO 710
C          BE RANGE LIMITED. IF THE TRACK IS TO BE LIMITED BY THE NUMBER 720
C          OF SAMPLES, THEN TRKEND MAY BE SET TO A LARGE NEGATIVE VALUE. 730
C                                                                        740
C        THE EQUATION...MAX ACCELERATION = 2*A*(V**2)*((2*PI/XPER)**2).. 750
C        MAY BE USEFUL IN DETERMINING VALUES FOR V, A, AND XPER.         760
C                                                                        770
C                                                                        780
      READ (5,1) CPA,HO                                                  790
C                                                                        800
C        THE THIRD DATA CARD PROVIDED MUST CONTAIN VALUES FOR THE        810
C        FOLLOWING VARIABLES, IN THE ORDER LISTED...(FORMAT 8F10.2.)     820
C                                                                        830
C          CPA = THE DISPLACEMENT IN THE Y-DIRECTION WHEN X=0.0 (YARDS). 840
C                                                                        850
C          HO = THE DISPLACEMENT IN THE Z-DIRECTION WHEN X=0.0 (YARDS).  860
C          THE DISPLACEMENT MUST BE IN THE POSITIVE Z-DIRECTION.         870
C                                                                        880
C        *** CAUTION *** THE USE OF NON-ZERO VALUES FOR CPA AND/OR HO    890
C        WILL RESULT IN A SHIFT IN THE TARGET POSITION WHICH MAY BE      900
C        UNDESIRABLE. FOR EXAMPLE, IF THE RANGES SPECIFIED FOR RMAN AND  910
C        TRKEND ARE CRITICAL THEN SPECIAL PRECAUTIONS MUST BE TAKEN IF   920
C        CPA OR HO IS NON-ZERO.                                          930
C                                                                        940
C                                                                        950
      READ (5,2) NSAM,N,ICOORD,IPUNCH                                    960
```

```
    2 FORMAT (4I10)                                                       970
C                                                                         980
C                                                                         990
C        THE FOURTH DATA CARD PROVIDED MUST CONTAIN VALUES FOR THE       1000
C        FOLLOWING FLAGS AND INTEGER VARIABLES IN THE ORDER LISTED.....  1010
C        THE FORMAT IS 4I10.                                             1020
C                                                                        1030
C        NSAM = THE NUMBER OF SAMPLES TO BE GENERATED IF THE TRACK IS TO 1040
C        BE LIMITED BY THE NUMBER OF SAMPLES. IF THE TRACK IS TO BE      1050
C        RANGE LIMITED, SET NSAM TO SOME LARGE VALUE (E.G. NSAM = 200).  1060
C                                                                        1070
C        N = THE NUMBER OF STATES TO BE GENERATED. N = 6 FOR A TRACK IN  1080
C        THE X-Y (RANGE-BEARING) PLANE. N = 9 FOR A 3-DIMENSIONAL TRACK. 1090
C                                                                        1100
C        ICOORD IS A FLAG. IF ICOORD IS NOT ZERO, THE TRACK OUTPUT WILL  1110
C        BE IN SPHERICAL COORDINATES. THE SPHERICAL COORDINATE OUTPUT     1120
C        HAS UNITS OF YARDS IN RANGE AND RADIANS IN BEARING AND ELEV.    1130
C                                                                        1140
C        IPUNCH IS A FLAG. IF IPUNCH IS NOT ZERO THE TRAJECTORY WILL BE  1150
C        PUNCHED ON DATA CARDS.                                          1160
C        THE PUNCHED CARD OUTPUT WILL BE DETERMINED BY STATEMENTS        1170
C        IMMEDIATELY PRECEDING STATEMENT NUMBER 17 OF THIS PROGRAM.      1180
C                                                                        1190
      NT = 1                                                             1200
      K = 1                                                              1210
C                                                                        1220
C          THE FOLLOWING SECTION GENERATES A STRAIGHT SECTION OF TRACK   1230
C        BEGINNING AT S(1,1) AND ENDING AT RMAN (OR WHEN K = NSAM IF A   1240
C        STRAIGHT TRACK IS DESIRED).                                     1250
C                                                                        1260
    3 K = K+1                                                            1270
      M = K-1                                                            1280
C                                                                        1290
C                                                                        1300
C                                                                        1310
      DO 4 I=3,6                                                         1320
    4 S(I,K) = 0.0                                                       1330
C                                                                        1340
C                                                                        1350
C                                                                        1360
      S(2,K) = -V                                                        1370
      S(1,K) = S(1,M)+S(2,M)*RATE                                        1380
      IF((K.GE.NSAM).OR.(S(1,K).LE.TRKEND)) GO TO 10                     1390
      IF (S(1,K).LE.RMAN) GO TO 5                                        1400
      GO TO 3                                                            1410
C                                                                        1420
C          THE FOLLOWING SECTION OF THE PROGRAM GENERATES THE SIN**2     1430
C          MANEUVER, BEGINNING AT RMAN AND HAVING A DURATION = XPER.     1440
```

```
C                                                                       1450
    5 Y(1) = XPER                                                       1460
      Y(2) = S(2,K)                                                     1470
      Y(3) = S(4,K)                                                     1480
      Y(4) = S(5,K)                                                     1490
      X = 0.0                                                           1500
      H = 0.01                                                          1510
      SAMP = 0.0                                                        1520
      B = 6.283185308/XPER                                              1530
      KHOLD = K                                                         1540
      NT = 0                                                            1550
    6 COS1 = DCOS(B*Y(1))                                               1560
      SIN1 = DSIN(B*Y(1))                                               1570
      COSSQ = COS1**2                                                   1580
      SINSQ = SIN1**2                                                   1590
      NSIGN = 2.0*Y(1)/XPER                                             1600
      P = (-1.0)**NSIGN                                                 1610
      F(1) = Y(2)                                                       1620
      F(2) = -4.*(A**2)*(B**3)*Y(2)*V*SIN1*COS1*(1.-2.*COSSQ)/(1.+4.*(A* 1630
     1*2)*(B**2)*COSSQ*SINSQ)**1.50                                     1640
      F(3) = Y(4)                                                       1650
      F(4) = 2.0*P*A*B*(F(2)*COS1*SIN1-B*(Y(2)**2)*(1.0-2.*COSSQ))      1660
      IF ((RATE-SAMP).GT.0.0001) GO TO 7                                1670
      K = K+1                                                           1680
      S(1,K) = S(1,KHOLD)+Y(1)-XPER                                     1690
      S(2,K) = Y(2)                                                     1700
      S(3,K) = F(2)                                                     1710
      S(4,K) = Y(3)                                                     1720
      S(5,K) = Y(4)                                                     1730
      S(6,K) = F(4)                                                     1740
      IF (K.GE.NSAM) GO TO 10                                           1750
      IF (Y(1).LE.0.0) GO TO 8                                          1760
      SAMP = 0.0                                                        1770
    7 CONTINUE                                                          1780
      G = RKLDEQ(4,Y,F,X,H,NT)                                          1790
      IF (G.LT.1.5) GO TO 6                                             1800
      SAMP = SAMP+H                                                     1810
      GO TO 6                                                           1820
CCC                                                                     1830
CCCCC                                                                   1840
CCCCC     THE FOLLOWING SECTION GENERATES A STRAIGHT SECTION OF TRACK.  1850
CCCCC     WHICH EXTENDS FROM THE COMPLETION OF THE MANEUVER TO TRKEND.  1860
CCC                                                                     1870
C                                                                       1880
    8 K = K+1                                                           1890
      M = K-1                                                           1900
CC                                                                      1910
CC                                                                      1920
```

```
C                                                                       1930
      DO 9 I=3,6                                                        1940
    9 S(I,K) = 0.0                                                      1950
C                                                                       1960
CC                                                                      1970
C                                                                       1980
      S(2,K) = -V                                                       1990
      S(1,K) = S(1,M)+S(2,M)*RATE                                       2000
      IF (K.GE.NSAM) GO TO 10                                           2010
      IF (S(1,K).LE.TRKEND) GO TO 10                                    2020
      GO TO 8                                                           2030
C                                                                       2040
CC         THE FOLLOWING SECTION ROTATES THE TRACK THROUGH THE ANGLES BR 2050
CC       AND E AS REQUIRED BY THE INPUT DATA.                           2060
C                                                                       2070
   10 B = BR/57.29577951                                                2080
      E = E/57.29577951                                                 2090
      COSB = DCOS(B)                                                    2100
      SINB = DSIN(B)                                                    2110
      COSE = DCOS(E)                                                    2120
      SINE = DSIN(E)                                                    2130
C                                                                       2140
CC                                                                      2150
C                                                                       2160
      DO 13 KL=1,K                                                      2170
C                                                                       2180
CC                                                                      2190
C                                                                       2200
      DO 11 JL=1,6                                                      2210
   11 TEMPL(JL) = S(JL,KL)                                              2220
C                                                                       2230
C                                                                       2240
      IF (N.NE.9) GO TO 12                                              2250
      S(7,KL) = TEMPL(1)*SINE +H0                                       2260
      S(8,KL) = TEMPL(2)*SINE                                           2270
      S(9,KL) = TEMPL(3)*SINE                                           2280
   12 IF (N.NE.9) COSE = 1.0                                            2290
C                                                                       2300
      R = DSQRT(TEMPL(1)**2+TEMPL(4)**2)                                2310
      THETA = DATAN2(TEMPL(4),TEMPL(1))                                 2320
      PHI = THETA+B                                                     2330
      S(1,KL) = R*DCOS(PHI)*COSE                                        2340
      S(2,KL) = (TEMPL(2)*COSB-TEMPL(5)*SINB)*COSE                      2350
      S(3,KL) = (TEMPL(3)*COSB-TEMPL(6)*SINB)*COSE                      2360
      S(4,KL) = R*COSE*DSIN(PHI)+CPA                                    2370
      S(5,KL) = TEMPL(5)*COSB+TEMPL(2)*SINB*COSE                        2380
      S(6,KL) = TEMPL(6)*COSB+TEMPL(3)*SINB*COSE                        2390
   13 CONTINUE                                                          2400
```

```
C                                                                       2410
CC                                                                      2420
CC                                                                      2430
C        CALL SUBROUTINE ICOORD IF SPHERICAL COORDINATE DATA IS DESIRED. 2440
C                                                                       2450
      IF (ICOORD.EQ.0) GO TO 14                                         2460
      CALL COORD (S,N,K)                                                2470
C                                                                       2480
   14 IF (ICOORD.EQ.0) WRITE (6,15)                                     2490
      IF (ICOORD.NE.0) WRITE (6,16)                                     2500
   15 FORMAT ('1',' X-POSITION  X-VELOCITY  X-ACCELER   Y-POSITION  Y-VE 2510
     1LOCITY  Y-ACCELER   Z-POSITION  Z-VELOCITY  Z-ACCELER ',/)        2520
   16 FORMAT ('1',5X,'RANGE',5X,'RDOT',8X,'R2DOT',8X,'BEARING',5X,'BDOT' 2530
     1,8X,'B2DOT',5X,'ELEVATION',5X,'EDOT',8X,'E2DOT',/)                2540
C                                                                       2550
CC                                                                      2560
C                                                                       2570
      DO 19 J=1,K                                                       2580
C                                                                       2590
CC       ***************************************************************2600
      IF (IPUNCH.EQ.0) GO TO 18                                         2610
C                                                                       2620
CC       WRITE STATEMENTS FOR PUNCHED CARD DATA OUTPUT MAY BE ADDED     2630
C        HERE.                                                          2640
      WRITE (7,17) S(1,J),S(2,J),S(3,J),S(4,J)                          2650
      WRITE (7,17) S(5,J),S(6,J),S(7,J),S(8,J)                          2660
      WRITE (7,17) S(9,J)                                               2670
C                                                                       2680
   17 FORMAT (4(1PD20.12))                                              2690
   18 CONTINUE                                                          2700
C                                                                       2710
CC       ***************************************************************2720
C                                                                       2730
      PK(J) = J                                                         2740
      PX(J) = S(1,J)                                                    2750
      PY(J) = S(4,J)                                                    2760
C                                                                       2770
CC                                                                      2780
   19 WRITE (6,20) (S(I,J),I=1,N),J                                     2790
C                                                                       2800
CC                                                                      2810
C                                                                       2820
   20 FORMAT (' ',9G12.4,5X,I5)                                         2830
      IF (NT.EQ.1) GO TO 23                                             2840
      WRITE (6,21)                                                      2850
   21 FORMAT ('1')                                                      2860
      CALL PLOTP (PX,PY,K,0)                                            2870
      WRITE (6,22)                                                      2880
```

```
   22 FORMAT ('0',T50,' Y(X,T)  VS X(T) ')                                  2890
   23 STOP                                                                  2900
      END                                                                   2910
      SUBROUTINE COORD (S,N,NSAM)                                            10
C                                                                            20
CCC      THIS SUBROUTINE CONVERTS CARTESIAN COORDINATE TRAJECTORY            30
CCC      POINTS TO SPHERICAL COORDINATES.                                    40
C                                                                            50
      IMPLICIT REAL*8(A-H,O-Z)                                               60
      DIMENSION S(N,NSAM)                                                    70
C                                                                            80
CCC                                                                          90
C                                                                           100
      DO 2 K=1,NSAM                                                         110
      X = S(1,K)                                                            120
      Y = S(4,K)                                                            130
      Z = 0.0                                                               140
      VX = S(2,K)                                                           150
      VY = S(5,K)                                                           160
      VZ = 0.0                                                              170
      AX = S(3,K)                                                           180
      AY = S(6,K)                                                           190
      AZ = 0.0                                                              200
      IF (N.LT.9) GO TO 1                                                   210
      Z = S(7,K)                                                            220
      VZ = S(8,K)                                                           230
      AZ = S(9,K)                                                           240
    1 CONTINUE                                                              250
      R = DSQRT(X**2+Y**2+Z**2)                                             260
      B = DATAN2(Y,X)                                                       270
      E = DARSIN(Z/R)                                                       280
      RDOT = (VX*X+VY*Y+VZ*Z)/R                                             290
      XYSQ = X**2+Y**2                                                      300
      IF (XYSQ.LT.0.0001) XYSQ=0.0001                                       310
      BDOT = (X*VY-VX*Y)/XYSQ                                               320
      RTRZ = DSQRT(R**2-Z**2)                                               330
      IF (RTRZ.LT.0.0001) RTRZ=0.0001                                       340
      EDOT = (R*VZ-RDOT*Z)/(R*RTRZ)                                         350
      R2DOT = (R*(AX*X+VX**2+AY*Y+VY**2+AZ*Z+VZ**2)-RDOT*(VX*X+VY*Y+VZ*Z    360
     1))/R**2                                                               370
      B2DOT = (X*AY-AX*Y)/XYSQ-2*((X*VX+Y*VY)*(X*VY-VX*Y))/XYSQ**2          380
      E2DOT = (R*AZ-R2DOT*Z)/RTRZ-(RDOT*RTRZ**2+R*(RDOT*R-VZ*Z)*(R*VZ-RD    390
     1DT*Z))/(R*RTRZ**3)                                                    400
      S(1,K) = R                                                            410
      S(2,K) = RDOT                                                         420
      S(3,K) = R2DOT                                                        430
      S(4,K) = B                                                            440
      S(5,K) = BDOT                                                         450
```

```
      S(6,K) = B2DOT                                                    460
      IF (N.LT.9) GO TO 2                                               470
      S(7,K) = E                                                        480
      S(8,K) = EDOT                                                     490
      S(9,K) = E2DOT                                                    500
    2 CONTINUE                                                          510
C                                                                       520
CC                                                                      530
C                                                                       540
      WRITE (6,3)                                                       550
    3 FORMAT ('1',T30,' OUTPUT DATA IN SPHERICAL COORDINATES',/)         560
      RETURN                                                            570
      END                                                               580
```

APPENDIX D

ADAPTIVE FILTER SUBROUTINES

FORTRAN listings of the three adaptive filter subroutines are included in this appendix. Also included is a flowchart which is based on the first version of the switch-on-residuals adaptive filter subroutine, because there is essentially no difference in the program flow, there is no separate flowchart provided for the second version.

Switch-On-Range Adaptive Filter

```
      SUBROUTINE ADAPT                                                        10
C                                                                             20
CC       THIS IS THE SWITCH-ON-RANGE VERSION OF SUBROUTINE ADAPT.             30
C                                                                             40
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI             50
      COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,      60
     19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9     70
     2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),     80
     3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9     90
     4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),     100
     5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,IW,IEST,ND,NFIRE,NHIT     110
     6,NSCARE,ICOORD,IADAPT                                                  120
C                                                                             130
CC    THE USER MUST SUPLY STATEMENTS HERE TO CONTROL THE ADAPTIVE             140
CC    FILTER OPERATION.                                                       150
CCC                                                                           160
CCC                                                                           170
CCC      CALCULATE THE TARGET RANGE FROM THE CARTESIAN MEASUREMENTS.          180
C                                                                             190
      RNG = SQRT(Z(1)**2+Z(2)**2+Z(3)**2)                                     200
C                                                                             210
CCC      IF THE TARGET RANGE IS LESS THAN THE SWITCHING RANGE SWITCH TO        220
CCC      THE HIGH Q GAIN MATRIX.                                              230
C                                                                             240
      IF ((RNG.GT.3333.33).OR.(K.LT.3)) GO TO 2                               250
C                                                                             320
    1 GK(1,1) = 0.250569                                                      330
      GK(2,1) = 0.144287                                                      340
      GK(4,2) = 0.719631                                                      350
      GK(5,2) = 1.42079                                                       360
      GK(7,3) = 0.719631                                                      370
      GK(8,3) = 1.42079                                                       380
    2 RETURN                                                                  390
      END                                                                     400
```

Switch-On-Residuals Adaptive Filter No. 1

```
      SUBROUTINE ADAPT                                                    10
C        THIS SUBROUTINE CONTROLS THE ADAPTIVE SWITCHING OF THE FILTER    20
C        PARAMETERS IN ORDER TO MAKE THE FILTER MORE RESPONSIVE IN A      30
C        CHANGING SITUATION. THIS IS THE SWITCH-ON-RANGE ADAPTIVE         40
C        FILTER SUBROUTINE.                                               50
CC                                                                        60
C                                                                         70
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI         80
      COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,  90
     19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9 100
     2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9), 110
     3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9 120
     4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160), 130
     5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,IW,IEST,ND,NFIRE,NHIT 140
     6,NSCARE,ICOORD,IADAPT                                               150
      DIMENSION GKA(9,9,41)                                               160
C                                                                         170
C     THE USER MUST SUPLY STATEMENTS HERE TO CONTROL THE ADAPTIVE         180
C     FILTER OPERATION.                                                   190
C                                                                         200
      IF (K-3) 1,2,6                                                      210
C                                                                         220
C        THE FILTER IS NOT ADAPTIVE FOR THE FIRST THREE SAMPLES.          230
C                                                                         240
    1 RETURN                                                              250
    2 IF (ITER.GT.1) GO TO 5                                              260
C                                                                         270
C        ON THE FIRST ITERATION READ IN THE MANEUVERING FILTER GAIN       280
C        SCHEDULE.                                                        290
CC                                                                        300
C                                                                         310
      DO 3 KK=1,40                                                        320
C                                                                         330
      DO 3 I=1,N                                                          340
    3 READ (5,4) (GKA(I,J,KK),J=1,M)                                      350
C                                                                         360
    4 FORMAT (4F20.12)                                                    370
    5 CONTINUE                                                            380
      IFLAGX = 0                                                          390
      IFLAGY = 0                                                          400
      IFLAGZ = 0                                                          410
      IBX = 0                                                             420
      IBY = 0                                                             430
      IBZ = 0                                                             440
C                                                                         450
C        SET IN THE STEADY STATE THEORETICAL VARIANCE OF POSITION         460
C        ESTIMATE ERROR VALUES TO BE USED IN CALCULATING THE GATING       470
C        LEVELS.                                                          480
```

```
C                                                                       490
      PX = 2.655313                                                     500
      PY = 2.93339                                                      510
      PZ = 2.93339                                                      520
C                                                                       550
CC       IF THE GATING FLAGS ARE ZERO RESET THE MANEUVERING GAIN        560
CC       SCHEDULE INDICES TO THEIR INITIAL VALUES.
C                                                                       580
    6 IF (IFLAGX.EQ.0) KX=0                                             530
      IF (IFLAGY.EQ.0) KY=0                                             540
      IF (IFLAGZ.EQ.0) KZ=0                                             590
      RSQ = Z(1)**2+Z(2)**2+Z(3)**2                                     610
      SINSQE = Z(3)**2/RSQ                                              620
      COSSQE = 1.0-SINSQE                                               630
      SIGSQR = SIGV(1)**2                                               640
      SIGSQB = SIGV(2)**2                                               650
      SIGSQE = SIGV(3)**2                                               660
C                                                                       670
CC                                                                      680
CC       CALCULATE THE VARIANCE OF MEASUREMENT NOISE USING EQ.(2.14)    690
C                                                                       700
      RKSQX = SIGSQR+RSQ*SIGSQE*SINSQE                                  710
      RKSQY = SIGSQB*RSQ*(COSSQE+SIGSQE*SINSQE)                         720
      RKSQZ = SIGSQE*RSQ*(COSSQE+SIGSQR*SINSQE)                         730
CC                                                                      740
CC       CALCULATE THE GATING LEVELS.                                   750
C                                                                       760
      BGX = PX+RKSQX                                                    770
      GATEX = 4.0*BGX                                                   780
      BGY = PY+RKSQY                                                    790
      GATEY = 4.0*BGY                                                   800
      BGZ = PZ+RKSQZ                                                    810
      GATEZ = 4.0*BGZ                                                   820
C                                                                       830
CC       CALCULATE THE SAMPLE RESIDUALS.                                840
C                                                                       850
      RESX = (Z(1)-XHKKM1(1))**2                                        860
      RESY=(Z(2)-XHKKM1(4   ))**2
      RESZ=(Z(3)-XHKKM1(  7   ))**2
      IF (RESX.LT.GATEX) GO TO 7                                        890
C                                                                       900
CC       IF THE X-POSITION RESIDUAL IS GREATER THAN THE X-POSITION      910
CC       GATING LEVEL SET GATING FLAG IFLAGX.
C                                                                       930
      IFLAGX = IFLAGX+1                                                 940
      IBX = 0                                                           950
      GO TO 8                                                           960
C                                                                       970
```

```
C          IF THE X-POSITION RESIDUAL IS LESS THAN ONE HALF THE GATING          980
C          LEVEL SET RESET FLAG IBX.
C                                                                              1000
    7 IF (RESX.LT.BGX) IBX=IBX+1                                               1010
C                                                                              1020
    8 IF (RESY.LT.GATEY) GO TO 9                                               1030
C                                                                              1040
C          IF THE Y-POSITION RESIDUAL IS GREATER THAN THE Y-POSITION           1050
C          GATING LEVEL SET GATING FLAG IFLAGY.
C                                                                              1070
      IFLAGY = IFLAGY+1                                                        1080
      IBY = 0                                                                  1090
      GO TO 10                                                                 1100
C                                                                              1110
C          IF THE Y-POSITION RESIDUAL IS LESS THAN ONE HALF THE GATING         1120
C          LEVEL SET RESET FLAG IBY.
C                                                                              1140
    9 IF (RESY.LT.BGY) IBY=IBY+1                                               1150
   10 IF (RESZ.LT.GATEZ) GO TO 11                                              1160
C                                                                              1170
C          IF THE Z-POSITION RESIDUAL IS GREATER THAN THE Z-POSITION           1180
C          GATING LEVEL SET GATING FLAG IFLAGZ.
C                                                                              1200
      IFLAGZ = IFLAGZ+1                                                        1210
      IBZ = 0                                                                  1220
      GO TO 12                                                                 1230
C                                                                              1240
C          IF THE Z-POSITION RESIDUAL IS LESS THAN ONE HALF THE GATING         1250
C          LEVEL SET RESET FLAG IBZ.
C                                                                              1270
   11 IF (RESZ.LT.BGZ) IBZ=IBZ+1                                               1280
C                                                                              1290
C          IF A RESIDUAL HAS BEEN LESS THAN ONE HALF THE GATING LEVEL          1300
C          FOR THREE SUCCESSIVE SAMPLES RESET THE CORRESPONDING GATING         1310
C          FLAG TO ZERO.
C                                                                              1330
   12 IF (IBX.GE.3) IFLAGX=0                                                   1340
      IF (IBY.GE.3) IFLAGY=0                                                   1350
      IF (IBZ.GE.3) IFLAGZ=0                                                   1360
C                                                                              1370
C          IF THE X-DIRECTION GATING FLAG, IFLAGX IS GREATER THAN ZERO
C          FOR TWO SUCCESSIVE SAMPLES SWITCH TO THE MANEUVERING GAIN
C          SCHEDULE FOR THE X-DIRECTION ESTIMATES.
C                                                                              1410
      IF (IFLAGX.LE.1) GO TO 14                                                1420
      KX = KX+1                                                                1430
      IF (KX.GT.39) KX = 40                                                    1440
C                                                                              1450
```

```
      DO 13 I=1,N3                                                      1460
   13 GK(I,1) = GKA(I,1,KX)                                             1470
C                                                                       1480
CCCCC                                                                   1490
CCCCC                                                                   1500
CCCCC      IF THE Y-DIRECTION GATING FLAG, IFLAGY IS GREATER THAN ZERO
CCCCC      FOR TWO SUCCESSIVE SAMPLES SWITCH TO THE MANEUVERING GAIN
CCCCC      SCHEDULE FOR THE Y-DIRECTION ESTIMATES.
C
   14 IF (IFLAGY.LE.1) GO TO 16                                         1550
      KY = KY+1                                                         1560
      IF (KY.GT.39) KY = 40                                             1570
C                                                                       1580
      DO 15 I=4,6                                                       1590
   15 GK(I,2) = GKA(I,2,KY)                                             1600
C                                                                       1610
CCCCC                                                                   1620
CCCCC                                                                   1630
CCCCC      IF THE Z-DIRECTION GATING FLAG, IFLAGZ IS GREATER THAN ZERO
CCCCC      FOR TWO SUCCESSIVE SAMPLES SWITCH TO THE MANEUVERING GAIN
CCCCC      SCHEDULE FOR THE Z-DIRECTION ESTIMATES.
C                                                                       167
   16 IF (IFLAGZ.LE.1) GO TO 18                                         168
      KZ = KZ+1                                                         169
      IF (KZ.GT.39) KZ = 40                                             170
C                                                                       171
      DO 17 I=7,9                                                       172
   17 GK(I,3) = GKA(I,3,KZ)                                             173
C                                                                       174
   18 RETURN                                                            175
      END                                                               1760
```

Switch-On-Residuals Adaptive Filter No. 2

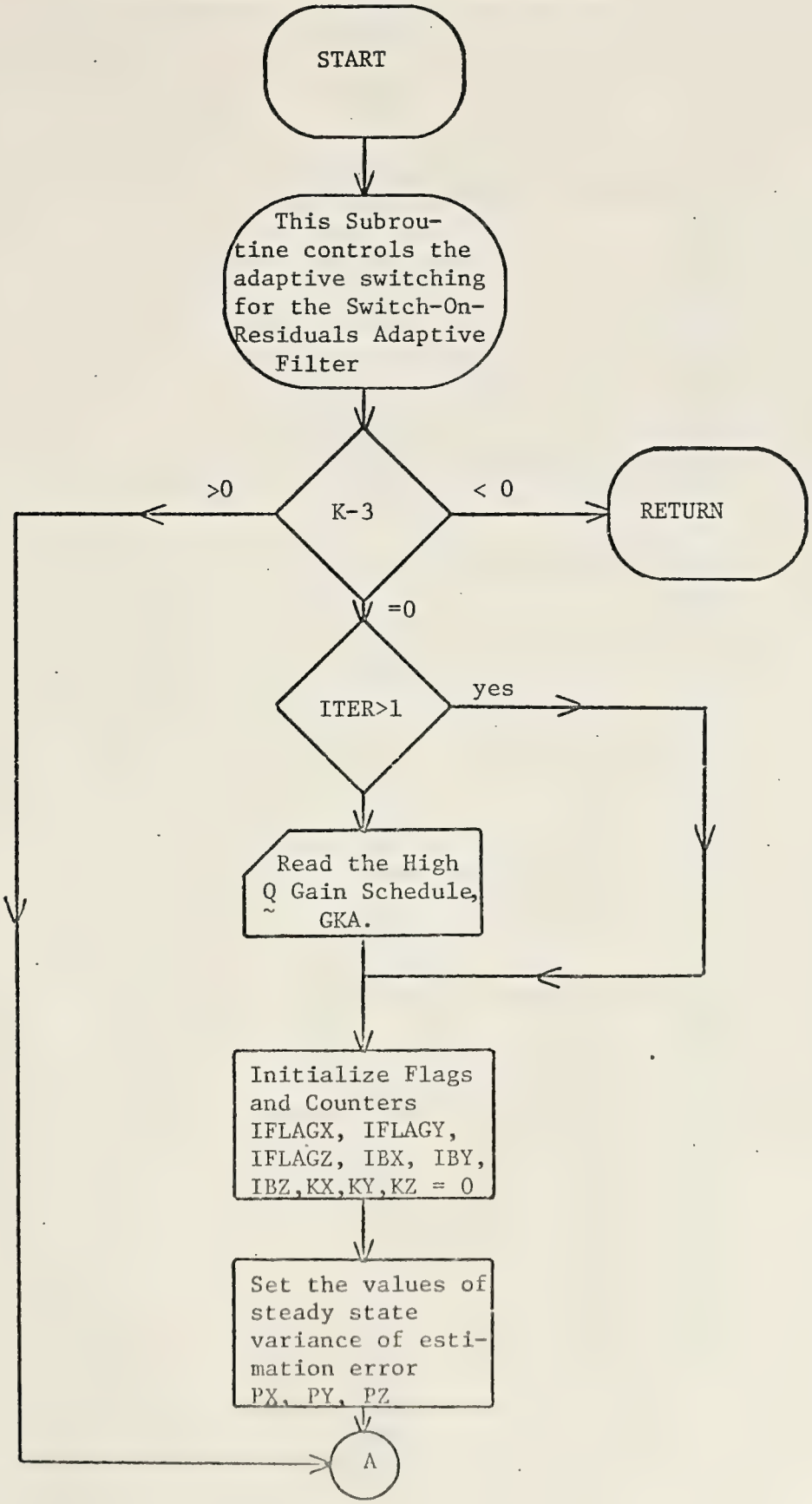
```
      SUBROUTINE ADAPT                                                      10
C        THIS SUBROUTINE CONTROLS THE ADAPTIVE SWITCHING OF THE FILTER      20
C        PARAMETERS IN ORDER TO MAKE THE FILTER MORE RESPONSIVE IN A        30
C        CHANGING SITUATION. THIS IS THE SWITCH-ON-RESIDUALS ADAPTIVE       40
C        FILTER SUBROUTINE.                                                 50
C                                                                           60
C                                                                           70
      REAL *8GAMMA,COVW,R,PHI,H,TEMP,TEMP1,TEMP2,PKKM1,G,PKK,Q,EI           80
      COMMON EI(9,9),Q(9,9),G(9,9),PKK(9,9),GAMMA(9,9),COVW(9,9),TEMP(9,    90
     19),TEMP1(9,9),TEMP2(9,9),H(9,9),PKKM1(9,9),R(9,9),PHI(9,9),VAR(9,9   100
     2,160),GKS(9,9,160),PKKS(9,9,160),XM(9,160),ERR(9,160),GAMMAS(9,9),   110
     3PHIS(9,9),XS(9,160),HS(9,9),GK(9,9),SIGW(9),X(9),SIGXZ(9),XZMEAN(9   120
     4),XHKK(9),XHKKM1(9),VTMP(9),Z(9),V(9),SIGV(9),XHATZ(9),AMISS(160),    130
     5N,NSAM,IQ,M,ITER,ITRK,IN,ISTAT,K,ITRO,IXZ,IV,IW,IEST,ND,NFIRE,NHIT   140
     6,NSCARE,ICCORD,IADAPT                                                150
      DIMENSION GKA(9,9,41)                                                160
C                                                                           170
C     THE USER MUST SUPLY STATEMENTS HERE TO CONTROL THE ADAPTIVE           180
C     FILTER OPERATION.                                                     190
C                                                                           200
      IF (K-13) 1,2,6                                                       210
C                                                                           220
C        THE FILTER IS NOT ADAPTIVE FOR THE FIRST THIRTEEN SAMPLES.         230
C                                                                           240
    1 RETURN                                                                250
    2 IF (ITER.GT.1) GO TO 5                                                260
C                                                                           270
C        ON THE FIRST ITERATION READ IN THE MANEUVERING FILTER GAIN         280
C        SCHEDULE.                                                          290
C                                                                           300
C                                                                           310
      DO 3 KK=1,40                                                          320
C                                                                           330
      DO 3 I=1,N                                                            340
    3 READ (5,4) (GKA(I,J,KK),J=1,M)                                        350
C                                                                           360
    4 FORMAT (4F20.12)                                                      370
    5 CONTINUE                                                              380
      IFLAGX = 0                                                            390
      IFLAGY = 0                                                            400
      IFLAGZ = 0                                                            410
      IBX = 0                                                               420
      IBY = 0                                                               430
      IBZ = 0                                                               440
      KX = 0                                                                450
      KY = 0                                                                460
      KZ = 0                                                                470
C                                                                           480
```

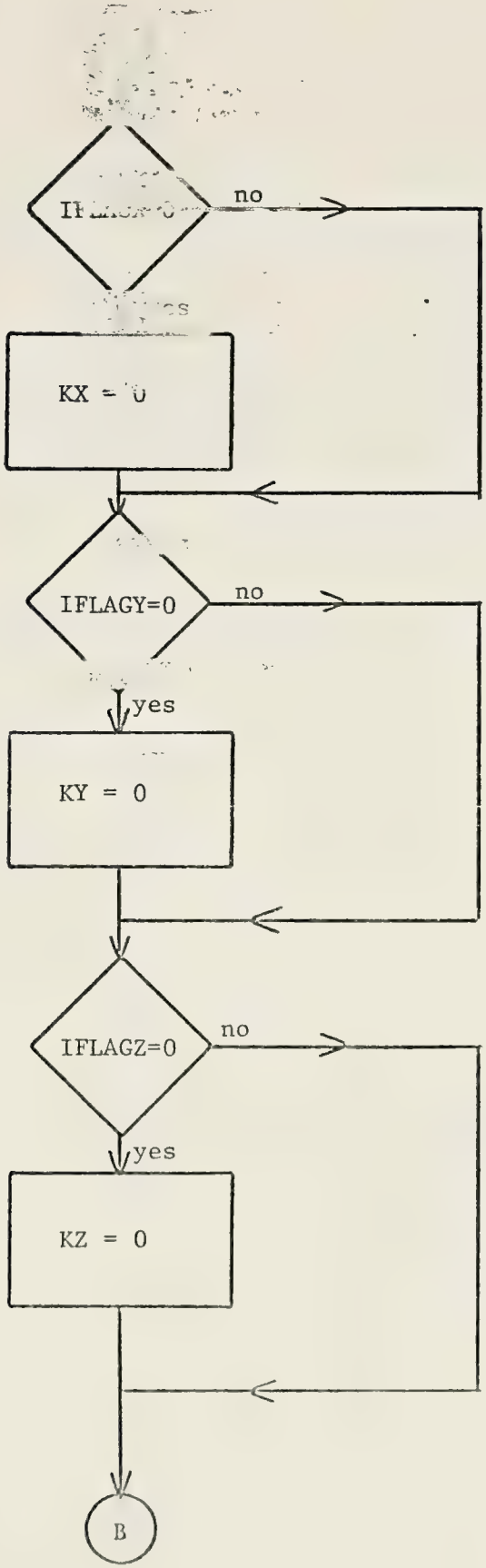
```
C          SET IN THE STEADY STATE THEORETICAL VARIANCE OF POSITION          490
CC         ESTIMATE ERROR VALUES TO BE USED IN CALCULATING THE GATING        500
CC         LEVELS.                                                           510
C                                                                            520
      PX = 2.655313                                                          530
      PY = 2.93339                                                           540
      PZ = 2.93339                                                           550
C                                                                            560
CC         IF THE GATING FLAGS ARE ZERO RESET THE MANEUVERING GAIN           570
CC         SCHEDULE INDICES TO THEIR INITIAL VALUES.                         580
C                                                                            590
    6 IF ((IFLAGX.EQ.0).AND.(IBX.GT.3)) KX=0                                 600
      IF ((IFLAGY.EQ.0).AND.(IBY.GT.3)) KY=0                                 610
      IF ((IFLAGZ.EQ.0).AND.(IBZ.GT.3)) KZ=0                                 620
      RSQ = Z(1)**2+Z(2)**2+Z(3)**2                                          630
      SINSQE = Z(3)**2/RSQ                                                   640
      COSSQE = 1.0-SINSQE                                                    650
      SIGSQR = SIGV(1)**2                                                    660
      SIGSQB = SIGV(2)**2                                                    670
      SIGSQE = SIGV(3)**2                                                    680
C                                                                            690
CCC                                                                          700
CCC        CALCULATE THE VARIANCE OF MEASUREMENT NOISE USING EQ.(2.14)       710
C                                                                            720
      RKSQX = SIGSQR+RSQ*SIGSQE*SINSQE                                       730
      RKSQY = SIGSQB*RSQ*(COSSQE+SIGSQE*SINSQE)                              740
      RKSQZ = SIGSQE*RSQ*(COSSQE+SIGSQR*SINSQE)                              750
C                                                                            760
CC         CALCULATE THE GATING LEVELS.                                      770
C                                                                            780
      BGX = PX+RKSQX                                                         790
      GATEX = 9.0*BGX                                                        800
      BGY = PY+RKSQY                                                         810
      GATEY = 9.0*BGY                                                        820
      BGZ = PZ+RKSQZ                                                         830
      GATEZ = 9.0*BGZ                                                        840
C                                                                            850
      BGX = 7.0*BGX                                                          860
      BGY = 7.0*BGY                                                          870
      BGZ = 7.0*BGZ                                                          880
C          CALCULATE THE SAMPLE RESIDUALS.                                   890
CC                                                                           900
      RESX = (Z(1)-XHKKM1(1))**2                                             910
      RESY = (Z(2)-XHKKM1(4))**2                                             920
      RESZ = (Z(3)-XHKKM1(7))**2                                             930
      IF (RESX.LT.GATEX) GO TO 7                                             940
C                                                                            950
CC         IF THE X-POSITION RESIDUAL IS GREATER THAN THE X-POSITION         960
```

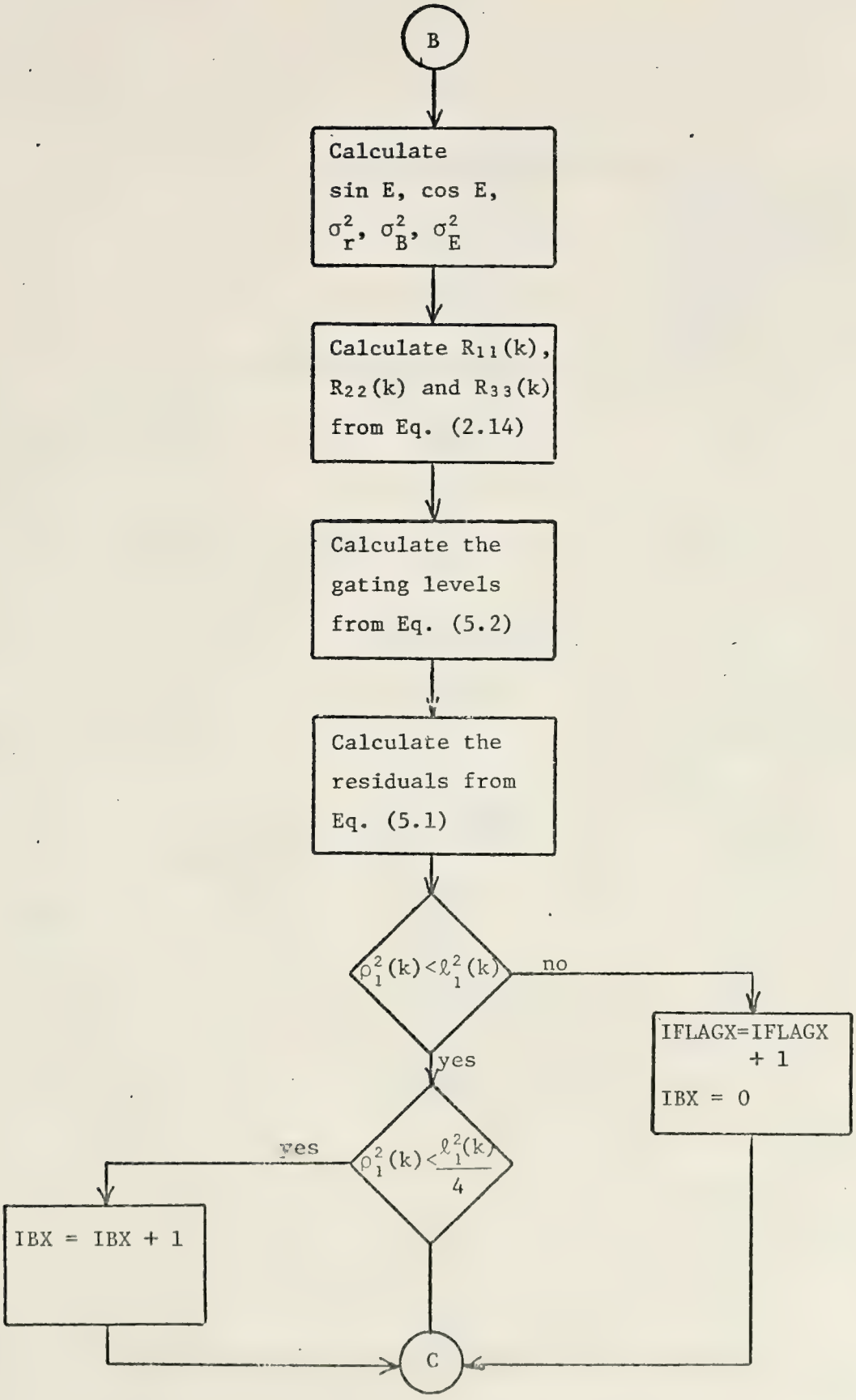
```
C        GATING LEVEL SET GATING FLAG IFLAGX.                                  970
CC                                                                             980
      IFLAGX = IFLAGX+1                                                        990
      IBX = 0                                                                 1000
      GO TO 8                                                                 1010
C                                                                             1020
CCC      IF THE X-POSITION RESIDUAL IS LESS THAN 88.0/100 THE GATING          1030
CCC      LEVEL SET RESET FLAG IBX.                                            1040
C                                                                             1050
    7 IF (RESX.LT.BGX) IBX=IBX+1                                              1060
C                                                                             1070
    8 IF (RESY.LT.GATEY) GO TO 9                                              1080
C                                                                             1090
CCC      IF THE Y-POSITION RESIDUAL IS GREATER THAN THE Y-POSITION            1100
CCC      GATING LEVEL SET GATING FLAG IFLAGY.                                 1110
C                                                                             1120
      IFLAGY = IFLAGY+1                                                       1130
      IBY = 0                                                                 1140
      GO TO 10                                                                1150
C                                                                             1160
CCC      IF THE Y-POSITION RESIDUAL IS LESS THAN 88.0/100 THE GATING          1170
CCC      LEVEL SET RESET FLAG IBY.                                            1180
C                                                                             1190
    9 IF (RESY.LT.BGY) IBY=IBY+1                                              1200
   10 IF (RESZ.LT.GATEZ) GO TO 11                                             1210
C                                                                             1220
CCC      IF THE Z-POSITION RESIDUAL IS GREATER THAN THE Z-POSITION            1230
CCC      GATING LEVEL SET GATING FLAG IFLAGZ.                                 1240
C                                                                             1250
      IFLAGZ = IFLAGZ+1                                                       1260
      IBZ = 0                                                                 1270
      GO TO 12                                                                1280
C                                                                             1290
CCC      IF THE Z-POSITION RESIDUAL IS LESS THAN  88/100  THE GATING          1300
CCC      LEVEL SET RESET FLAG IBZ.                                            1310
C                                                                             1320
   11 IF (RESZ.LT.BGZ) IBZ=IBZ+1                                              1330
C                                                                             1340
CCC      IF A RESIDUAL HAS BEEN LESS THAN ONE HALF THE GATING LEVEL           1350
CCC      FOR TWO   SUCCESSIVE SAMPLES RESET THE CORRESPONDING GATING          1360
CCC      FLAG TO ZERO.                                                        1370
C                                                                             1380
   12 IF (IBX.GE.2) IFLAGX=0                                                  1390
      IF (IBY.GE.2) IFLAGY=0                                                  1400
      IF (IBZ.GE.2) IFLAGZ=0                                                  1410
C                                                                             1420
CCC      IF THE X-DIRECTION GATING FLAG, IFLAGX IS GREATER THAN ZERO          1430
CC       SWITCH TO THE MANEUVERING GAIN SCHEDULE FOR  THE X-DIRECTION         1440
```

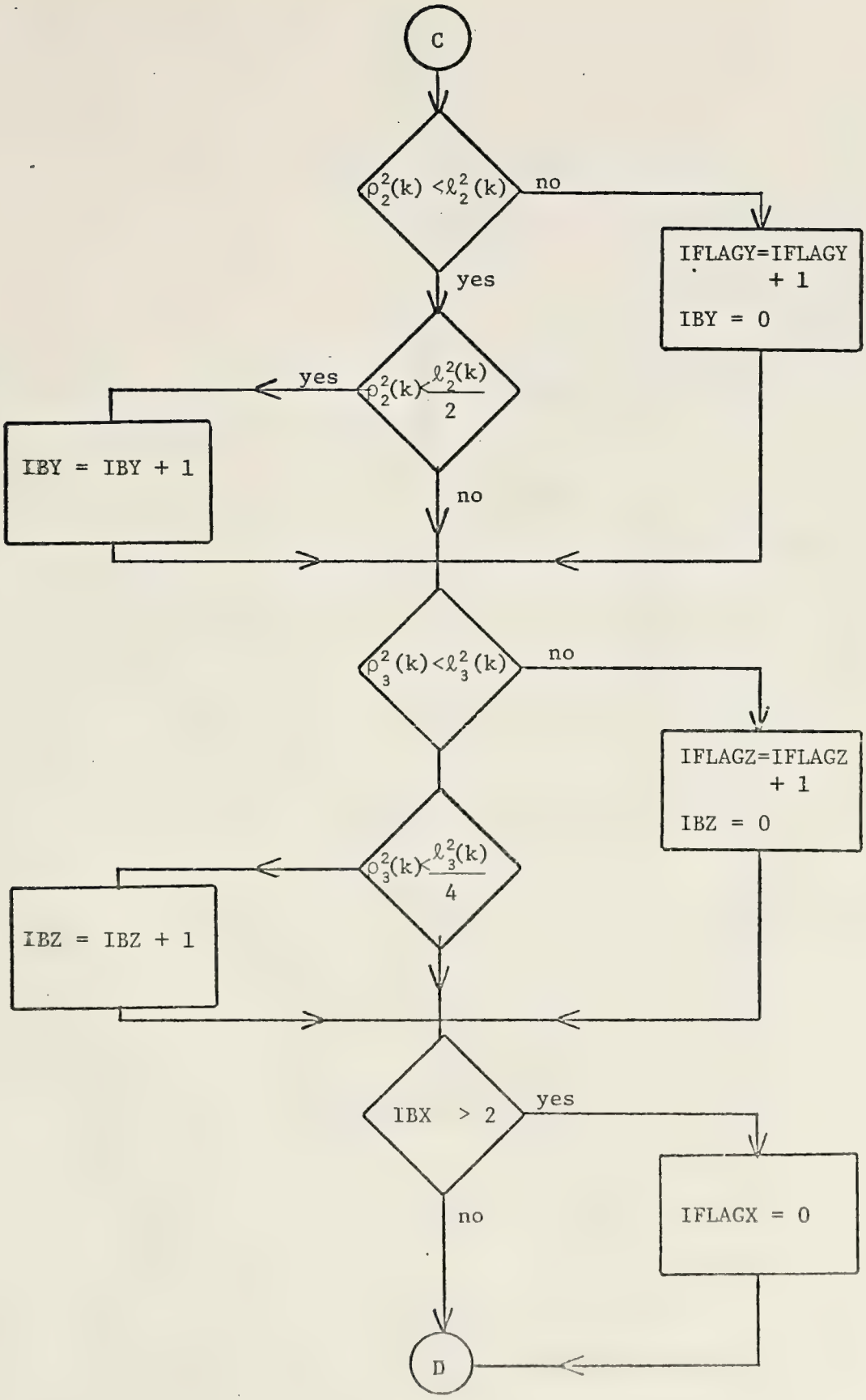
```
C        STATE ESTIMATES.                                                    1450
CC                                                                           1460
      IF (IFLAGX.LT.1) GO TO 14                                              1470
      KX = KX+1                                                              1480
      IF (KX.GT.39) KX = 40                                                  1490
C                                                                            1500
      DO 13 I=1,N3                                                           1510
   13 GK(I,1) = GKA(I,1,KX)                                                  1520
CC                                                                           1530
CCCC                                                                         1540
CCCC                                                                         1550
CCCC     IF THE Y-DIRECTION GATING FLAG, IFLAGY IS GREATER THAN ZERO         1560
CCCC     SWITCH TO THE MANEUVERING GAIN SCHEDULE FOR  THE Y-DIRECTION        1570
CCCC     STATE ESTIMATES.                                                    1580
CC                                                                           1590
   14 IF (IFLAGY.LT.1) GO TO 16                                              1600
      KY = KY+1                                                              1610
      IF (KY.GT.39) KY = 40                                                  1620
C                                                                            1630
      DO 15 I=4,6                                                            1640
   15 GK(I,2) = GKA(I,2,KY)                                                  1650
CC                                                                           1660
CCCC                                                                         1670
CCCC                                                                         1680
CCCC     IF THE Z-DIRECTION GATING FLAG, IFLAGZ IS GREATER THAN ZERO         1690
CCCC     SWITCH TO THE MANEUVERING GAIN SCHEDULE FOR  THE Z-DIRECTION        1700
CCCC     STATE ESTIMATES.                                                    1710
CC                                                                           1720
   16 IF (IFLAGZ.LT.1) GO TO 18                                              1730
      KZ = KZ+1                                                              1740
      IF (KZ.GT.39) KZ = 40                                                  1750
C                                                                            1760
      DO 17 I=7,9                                                            1770
   17 GK(I,3) = GKA(I,3,KZ)                                                  1780
C                                                                            1790
   18 RETURN                                                                 1800
      END                                                                    1810
```

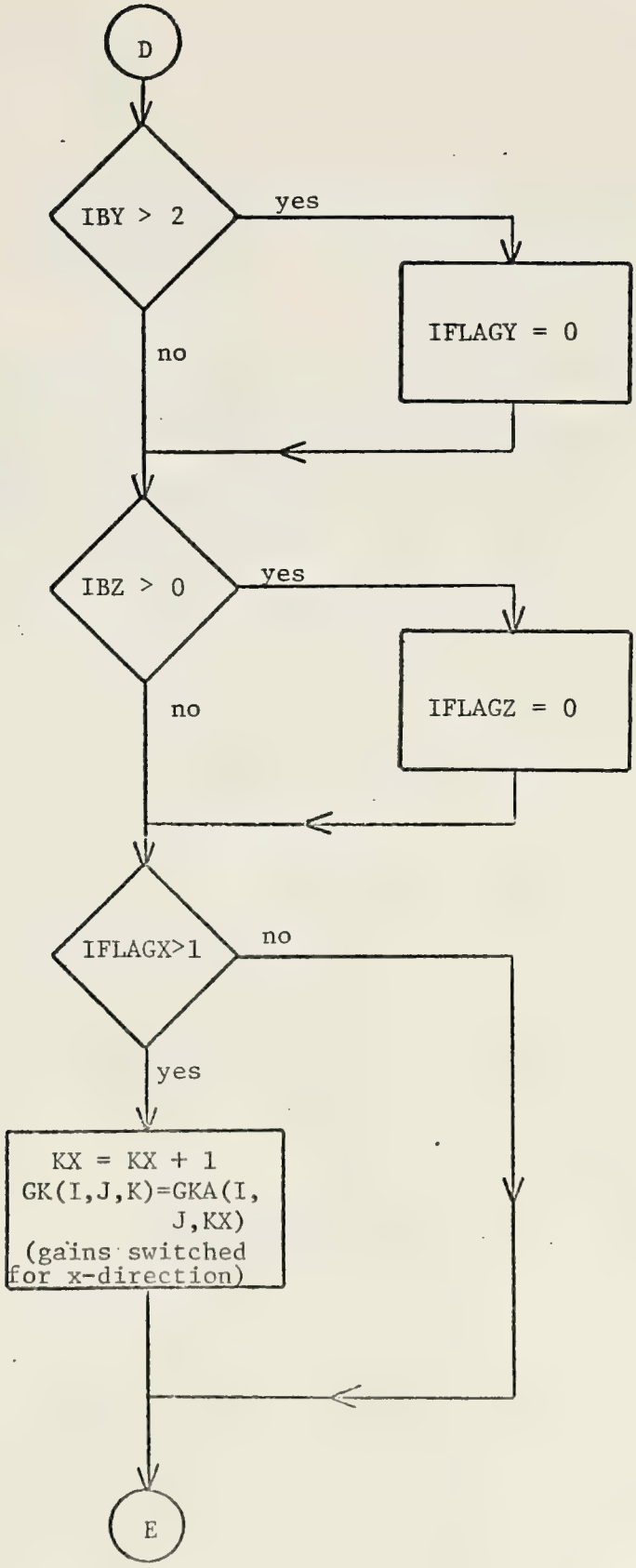
SWITCH-ON-RESIDUALS SUBROUTINE FLOWCHART

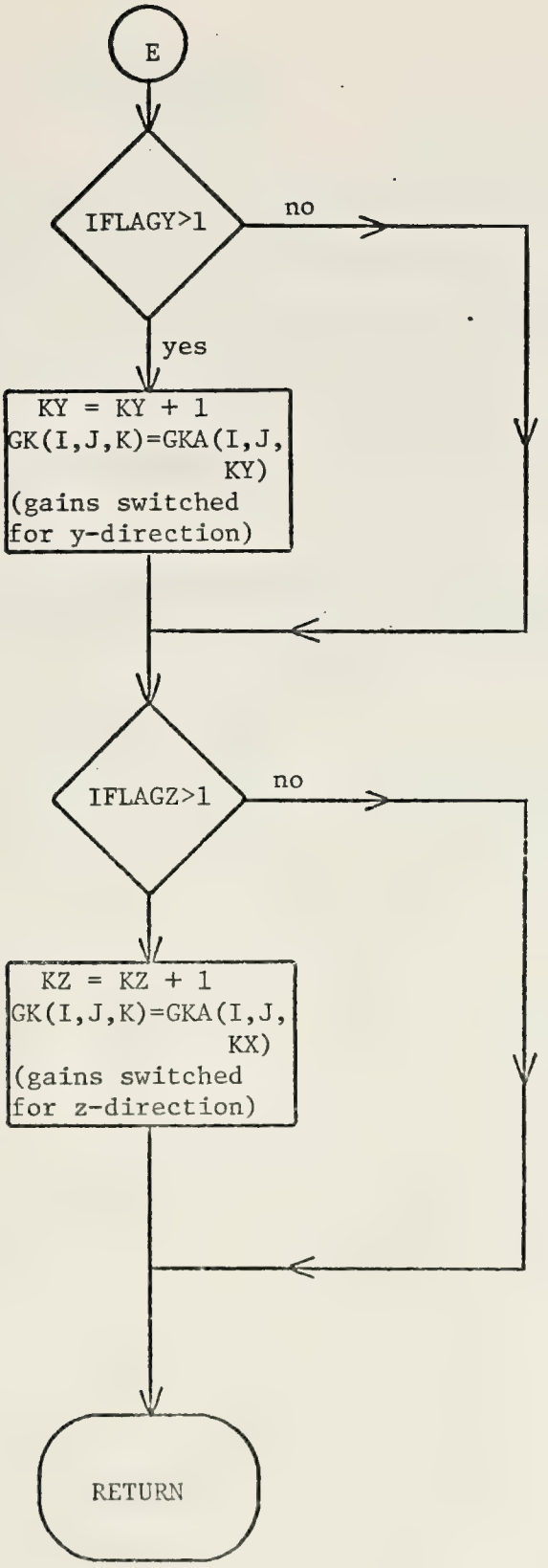

START
This Subroutine controls the adaptive switching for the Switch-On-Residuals Adaptive Filter
K-3
>0
< 0
RETURN
=0
ITER>1
yes
Read the High Q Gain Schedule, ~ GKA.
Initialize Flags and Counters IFLAGX, IFLAGY, IFLAGZ, IBX, IBY, IBZ,KX,KY,KZ = 0
Set the values of steady state variance of estimation error PX, PY, PZ
A


no
KX = 0
IFLAGY=0
no
yes
KY = 0
IFLAGZ=0
no
yes
KZ = 0
B


B
Calculate sin E, cos E, σ_r^2, σ_B^2, σ_E^2
Calculate $R_{11}(k)$, $R_{22}(k)$ and $R_{33}(k)$ from Eq. (2.14)
Calculate the gating levels from Eq. (5.2)
Calculate the residuals from Eq. (5.1)
$\rho_1^2(k) < \ell_1^2(k)$
no
yes
IFLAGX=IFLAGX + 1
IBX = 0
$\rho_1^2(k) < \frac{\ell_1^2(k)}{4}$
yes
IBX = IBX + 1
C


C
$\rho_2^2(k) < \ell_2^2(k)$
no
yes
IFLAGY=IFLAGY + 1
IBY = 0
yes
$\rho_2^2(k) < \frac{\ell_2^2(k)}{2}$
IBY = IBY + 1
no
$\rho_3^2(k) < \ell_3^2(k)$
no
IFLAGZ=IFLAGZ + 1
IBZ = 0
$\rho_3^2(k) < \frac{\ell_3^2(k)}{4}$
IBZ = IBZ + 1
IBX > 2
yes
IFLAGX = 0
no
D


D
IBY > 2
yes
IFLAGY = 0
no
IBZ > 0
yes
IFLAGZ = 0
no
IFLAGX>1
no
yes
KX = KX + 1
GK(I,J,K)=GKA(I,
J,KX)
(gains switched
for x-direction)
E


E
IFLAGY>1
no
yes
KY = KY + 1
GK(I,J,K)=GKA(I,J,
KY)
(gains switched
for y-direction)
IFLAGZ>1
no
KZ = KZ + 1
GK(I,J,K)=GKA(I,J,
KX)
(gains switched
for z-direction)
RETURN

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